26151904

26151904

Reg. No.....

Name.....

M.Sc. DEGREE (PRIVATE) EXAMINATION, JULY 2026

First Semester

Faculty of Science

Branch I (A) : Mathematics

Paper V—GRAPH THEORY

(2016—2018 Admissions—Final Mercy Chance)

Time : Three Hours | Maximum Marks : 75

Answer **either** *Part A* **or** *Part B of each question.*
Each question carries 15 *marks.*

I. A (i) Define a graphical sequence.

(ii) Prove that in any group of n persons $(n \geq 2)$, there are at least two with the same number of friends.

(iii) Define a bipartite graph and prove that a graph is bipartite if and only if it contains no odd cycles.

B (i) Define a directed graph.

(ii) Let D be a digraph with no directed cycle. Prove that there exists vertex whose in degree is zero.

(iii) Define a tournament, and prove that every tournament contains a directed Hamiltonian path.

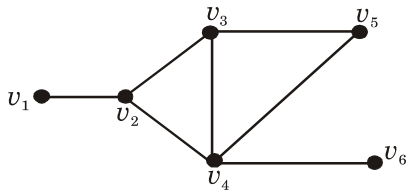
(iv) Show that every tournament T is disconnected or can be made into one by the reorientation of just one arc of T.

II. A (i) Prove that a vertex v of a connected graph G with at least three vertices is a cut vertex of G if and only if there exists vertices u and w of G distinct from v such that v is an every u-w path in G.

Turn over

(ii) Prove that a simple cubic connected graph G has a cut vertex if and only if it has a cut edge.

(iii) Find the vertex cuts and edge cub of the following graph.



B (i) Prove that the connectivity and edge connectivity of a simple cubic graph G are equal.

(ii) State and prove the max-flow min-cut theorem.

III. A (i) Prove that the number of edges in a tree on n vertices is $n-1$. Also prove that a connected graph on n-vertices and $n-1$ edges is a tree.

(ii) Prove that every tree has a centre consisting of either a single vertex or two adjacent vertices.

B (i) For the graph G given below, determine two minimum weight spanning trees using :

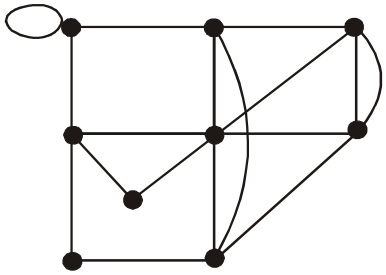
(i) Kruskal's algorithm ; and

(ii) Prim's algorithm.

(ii) Prove that a connected graph G is a tree if and only if every edge of G is a cut edge of G.

IV. A (i) Prove that a graph is Eulerian if and only if it has an odd number of cycle decompositions.

(ii) Find an Euler tour in the graph given below.



B (i) Determine the chromatic number of :

(i) The Petersen graph ; and

(ii) Wheel W_n.

(ii) Prove that in a critical graph, no vertex cut is an clique.

(iii) Prove that if G is triangle free, then $\mu(G)$ is also triangle free.

V. A (i) Prove that a graph is planar if and only if it is embeddable on a sphere.

(ii) Prove that a graph is planar if and only if each of its blocks is planar.

B (i) Prove that $K_{3,3}$ is non-planar.

(ii) Prove that every planar graph is 5-vertex colorable.

$(5 \times 15 = 75)$