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Reg. No.....

Name.....

M.Sc. DEGREE (PRIVATE) EXAMINATION, JULY 2026

First Semester

Faculty of Science

Branch I (A) : Mathematics

Paper III—TOPOLOGY – I

(2016 – 2018 Admissions–Final Mercy Chance)

Time : Three Hours Maximum Marks : 75

Answer **either** *Part A* **or** *Part B of each question.*

Each question carries 15 marks.

I. A i) Define a topological space.

ii) Show that intersection of a family of topologies on a set X is a topology on X.

iii) Is union of topologies on X is a topology on X. Justify your answer.

iv) Prove that if a space is second countable, then every open cover of it has a countable subcover.

B (i) Let X be a set and $\mathcal{B}$ a family of its subsets covering X. Prove that the following statements are equivalent.

a) There exists a topology on X with $\mathcal{B}$ as base.

b) For any $B_1, B_2 \in \mathcal{B}, x \in B_1 \cap B_2$ can be expressed as the union of some members of $\mathcal{B}$.

c) For any $B_1, B_2 \in \mathcal{B}$ and $x \in B_1 \cap B_2$, there exists $B_3 \in \mathcal{B}$ such that $x \in B_3$ and $B_3 \subset B_1 \cap B_2$

(ii) Define the terms second countability and metrisability and show that second countability is a hereditary property.

II. A i) Define a dense set in a topological space X and prove that a subset A of a space X is dense in X if and only if for every non empty open set B of X, $A \cap B \neq \phi$.

ii) Prove that in a metric space X, a point y is in the closure of a set A if and only if there exists a sequence $\{x_n\}$ such that $x_n \in A$ for all n and $\{x_n\}$ converges to $y \in X$.

iii) Let X be a space and $A \subset X$. Show that $\text{int}(A)$ is the union of all open sets contained in A and is also the largest open subset of X contained in A.

B i) Prove that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.

ii) For any subset A of a space X, prove that $\bar{A} = A \cup A'$, where A' denotes the derived set of A.

III. A i) Define a continuous function from a space X to a space Y. and give an example.

ii) Prove that composition of continuous functions are continuous.

iii) Let $\{(X_i, \Im_i)\}$ for $i = 1,2.....,n$ be an indexed collection of topological spaces and let $(X, \Im)$ be their product topology. Prove that each projection function π_i is continuous.

B i) Prove that in the case of finite products. the product topology is the weak topology determined by the projection functions.

ii) Define a quotient map and prove that every closed surjective map is a quotient map.

IV. A i) Define compact and Lindeloff spaces.

ii) Let $(X, \Im)$ be a topological space and $A \subset X$. Prove that A is a compact subset of X if and only if the subspace $(A, \Im/A)$ is compact.

iii) Define a separable space and show that every second countable space is separable.

B i) Prove that every continuous real valued function on a compact space is bounded and attains its extrema.

ii) Prove that every continuous image of a compact space is compact.

V. A i) Define a connected space and prove that continuous image of a connected space is connected.

ii) Prove that a subset of R is connected if and only if it is an interval.

B i) Define a locally connected space and prove that every quotient space of a locally connected space is locally connected.

ii) Define a path connected space and prove that every path connected space is connected.

(5 × 15 = 75)