

MAHATMA GANDHI UNIVERSITY, KOTTAYAM

MGU-UGP (HONOURS)

FIFTH SEMESTER EXAMINATION (2024 ADMISION ONWARDS)

MG5DSCMAT300: A First Course in Complex Analysis

Duration: 2 hrs

Maximum Marks: 70

Students should attempt at least one question from each course outcome to enhance their overall outcome attainability.

Part A

Answer any five questions. Each question carries 2 marks

1. Evaluate $\int_0^1 (1 + it)^2 dt$ [U][1]
2. State the Cauchy –Goursat theorem. [K][1]
3. Find the Maclaurian series for $f(z)=z\sin z$. [U][6]
4. Evaluate $\lim_{n \rightarrow \infty} \frac{n^2 + in}{n^2 + 1}$. [U][2]
5. Find the singular points, if any, of the function $\frac{\sin z^2}{z^2 + 1}$ [U][2]
6. State the formula for finding the residue of a function $f(z)$ at a pole of order m . [U][2]
7. Define improper integral of a continuous function $f(x)$ over the interval $-\infty < x < \infty$. [K][7]
8. State Jordan's inequality. [K][7]

Part B Answer any five questions. Each question carries 6 marks

9. State and prove the Principle of Deformation of Paths [K][2]
10. Evaluate $\int_C \frac{\cos z}{z(z^2 + 8)} dz$ where C is the circle $|z| = 3$. [A][3]
11. Using the formal definition of limit, prove that the sequence $z_n = i/n$ converges to i . [U][2]
12. Find Taylor series of $f(z) = 1/z^2$ about $z=2$ and specify the region in which series is valid. [A][6]
13. Find the residue at $z = 0$ of the function $\cot z / z^4$. [U][2]
14. Determine the nature of singularity of $f(z) = \frac{1}{z(e^z - 1)}$. [An][5]
15. Evaluate $\int_0^{2\pi} \frac{d\theta}{3 + 2\sin\theta}$. [E][7]
16. State Jordan's lemma [K][7]

Part C Answer any three questions. Each question carries 10 marks

17. State and prove Liouville's theorem.

[An][4]

18. (a) State Taylor's theorem.

(b) Show that $\frac{1+2z^2}{z^3+z^5} = \frac{1}{z^3} + \frac{1}{z} - z + z^3 - z^5 \dots$ ($0 < |z| < 1$)

[U][6]

19. Find Laurent series expansion for $f(z) = z^2 \sin(1/z^2)$ and $f(z) = z^3 \cos(1/z^3)$. [C][6]

20. Describe the different types of isolated singularities with examples. [U][2]

21. Use residues to find the Cauchy Principal value of the integral $\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$.

[E][7]

22. Show that $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$, $a > |b|$.

[E][7]

MAHATMA GANDHI UNIVERSITY, KOTTAYAM
MGU-UGP (HONOURS)
FIFTH SEMESTER EXAMINATION
(2024 ADMISION ONWARDS)
MG5DSCMAT301-LIMITS AND CONVERGENCE

Duration: 2 hrs

Maximum Marks: 70

Students should attempt atleast one question from each course outcome to enhance their overall outcome attainability

Part A

Answer any five questions. Each questions carries 2 marks

1. Define the tail of a sequence. Give an example to illustrate the definition. [U] [1]
2. State Monotone Convergence Theorem [K] [2]
3. Discuss the convergence of the sequence $\left(1, \frac{1}{2}, 3, \frac{1}{4}, \dots\right)$ [An][4]
4. When do you say that a sequence (x_n) is properly divergent? [K] [5]
5. Apply the Comparison Test to determine whether the series $\sum_{n=1}^{\infty} \left(\frac{1}{n^2+n}\right)$ converges or diverges. [A] [5]
6. State Raabe's Test for determining the convergence or divergence of a series. [K] [6]
7. Explain sequential criterion for limits of functions [U] [8]
8. Describe when a function $f(x)$ is said to be bounded in a neighbourhood of a cluster point. [U] [8]

Part B

Answer any five questions. Each questions carries 6 marks

9. Apply the definition of the limit of a sequence to show that a sequence in R can have at most one limit. [A] [1]
10. Prove that every convergent sequence of real numbers is bounded. [An][1]
11. State and prove Monotone Subsequence Theorem. [An][3]
12. Explain the nature of the sequence $\left(\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{n}\right)$. [U] [4]
13. Determine the nature of the series whose n th term is $\frac{n!}{3.5.7....(2n+1)}$. [A] [6]
14. Apply Dirichlet's Test to determine the convergence of $\sum_{n=1}^{\infty} (a_n \sin nx)$, where (a_n) is decreasing and converges to zero. [A] [7]
15. Discuss why $\lim_{x \rightarrow 0} \left(x \sin \left(\frac{1}{x}\right)\right)$ approaches 0 as $x \rightarrow 0$ [U] [8]
16. Describe, using the definition of limit at infinity, why $\lim_{x \rightarrow +\infty} x^n = +\infty, n \in N$. [U] [8]

Part C

Answer any three questions. Each questions carries 10 marks

17. State and prove squeeze theorem for sequences and hence find $\lim_{n \rightarrow \infty} \left(\frac{\sin n}{n}\right)$. [An][1]

18. a) Show that every convergent sequence is a Cauchy sequence. Is the converse true? Explain your answer.
 b) Show that a sequence of real numbers is convergent if and only if it is a Cauchy sequence. [An][4]
19. Let (x_n) and (y_n) be two sequences of positive real numbers and suppose that for some $L > 0$, $\lim(\frac{x_n}{y_n}) = L$. Apply the limit properties to show that $\lim(x_n) = +\infty$ if and only if $\lim(y_n) = +\infty$. [A] [5]
20. a) Define the rearrangement of a series and state rearrangement theorem on the absolute convergence.
 b) Show that if a series $\sum x_n$ is convergent, then any series obtained from it by grouping the terms is also convergent and to the same limit. [A] [6]
21. a) Explain i) a cluster point of a subset of R , ii) limit of a function at a cluster point.
 b) Describe how a cluster point of a subset of R can be characterized using a sequence in the subset. [U] [8]
22. a) Interpret whether the one-sided limits of the function $g(x) = e^{\frac{1}{x}}$, at $x = 0$ exists or not.
 b) Explain whether the function $f(x) = \text{sgn}(x)$ has distinct right-hand and left-hand limits at $x = 0$. [U] [8]

MAHATMA GANDHI UNIVERSITY, KOTTAYAM

MGU-UGP (HONOURS)

FIFTH SEMESTER EXAMINATION (2024 ADMISSION ONWARDS)

MG5DSCMAT302 – FUNDAMENTALS OF GROUPS AND RINGS

MODEL QUESTION PAPER

Duration: 2 Hours

Maximum Marks: 70

*Remember(K), Understand(U), Apply(A), Analyse(An), Evaluate(E),
Create(C)*

Students should attempt at least one question from each course outcome to enhance their overall outcome attainability.

Part A

Answer any **5** questions. Each question carries **2** marks.

1. Let $\cdot$ be the usual multiplication operation on $\mathbb{Z}$ the set of integers, and let $H = \{n^2 | n \in \mathbb{Z}^+\}$. Determine whether H is closed under multiplication. [U][1]
2. Find the identity element of the set $\mathbb{Z}$ with binary operation $*$ defined by $a * b = ab + a + b$. [U][1]
3. List the generators of (a) $\mathbb{Z}$ (b) $\mathbb{Z}_5$ [U][2]
4. Find the quotient and remainder, according to the division algorithm, when -42 is divided by 9. [U][2]
5. List the elements of the subgroup generated by $\{12, 30\}$ of $\mathbb{Z}_{36}$. [U][3]
6. State the theorem of Lagrange. [K][3]
7. Define ring homomorphism. [K][4]
8. Find the characteristic of the ring $\mathbb{Z}_6 \times \mathbb{Z}_{15}$. [K][4]

[2 × 5 = 10]

Part B

Answer any **5** questions. Each question carries **6** marks.

9. Show that Q^+ with the operation $*$ defined by $a * b = ab/2$ is a group. [U][1]

10. Show that $\langle n\mathbb{Z}, + \rangle$ is isomorphic to $\langle \mathbb{Z}, + \rangle$. [U][1]
11. Compute σ^{100} given $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$. [A][5]
12. Give the subgroup diagram of the subgroups of $\mathbb{Z}_{12}$. [U][2]
13. Find all orbits of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 3 & 6 & 2 & 4 \end{pmatrix}$. [A][3]
14. Find all cosets of the subgroup $4\mathbb{Z}$ of $\mathbb{Z}$. [U][3]
15. Determine whether $\mathbb{Z} \times \mathbb{Z}$ is a ring with addition and multiplication by components. [U][4]
16. Prove every finite integral domain is a field. [U][4]

[6 × 5 = 30]

Part C

Answer any **3** questions. Each question carries **10** marks.

17. (a) Show that the left and right cancellation laws hold in a group G with binary operation $*$.
(b) Prove that the identity element and inverse of each element are unique in a group. [U][1]
18. (a) Prove a subgroup of a cyclic group is cyclic.
(b) Describe all cyclic groups upto isomorphism. [U][2]
19. (a) Let G be a cyclic group with n elements and generated by a . Let $b \in G$ and let $b = a^s$. Prove that b generates a cyclic subgroup H of G containing n/d elements, where d is the greatest common divisor of n and s .
(b) Find the order of all the subgroups of $\mathbb{Z}_{18}$. [U][2]
20. Let ϕ be a homomorphism of a group G into a group G' . Prove
(a) If e is the identity element in G , then $\phi(e)$ is the identity element e' in G' .
(b) If $a \in G$, then $\phi(a^{-1}) = \phi(a)^{-1}$.
(c) If H is a subgroup of G , then $\phi(H)$ is a subgroup of G' .

(d) If K' is a subgroup of G' , then $\phi^{-1}(K')$ is a subgroup of G .

[U][3]

21. Let R be a ring with additive identity 0 , and let $a, b \in R$. Prove

(a) $0a = a0 = 0$

(b) $a(-b) = (-a)b = -(ab)$

(c) $(-a)(-b) = ab$.

[U][4]

22. (a) Solve the equation $x^2 - 5x + 6 = 0$ in $\mathbb{Z}_{12}$.

(b) Classify each nonzero element of $\mathbb{Z}_{20}$ as a unit or a 0 divisor.

(c) Prove the cancellation laws hold in a ring R if and only if R has no divisors of 0.

[A][4]

[10 × 3 = 30]

MAHATMA GANDHI UNIVERSITY, KOTTAYAM
MGU-UGP (HONOURS)
FIFTH SEMESTER EXAMINATION
(2024 ADMISION ONWARDS)
MG5DSEMAT300: DIFFERENTIAL EQUATIONS AND APPLICATIONS

Duration: 2 hrs

Maximum Marks: 70

Students should attempt atleast one question from each course outcome to enhance their overall outcome attainability.

Part A

Answer any five questions. Each question carries 2 marks

1. Define exact differential equations. [K][1]
2. Define Bernoulli differential equation [K][1]
3. What is mean by orthogonal trajectory of a family of curves. [K][2]
4. What is the integrating factor of the differential equation $M(x, y)dx + N(x, y)dy = 0$. [U][1]
5. State the basic existence theorem. [K][3]
6. What is the auxiliary equation of the nth order linear differential equation [U][3]
7. State Sonia Kowulewski's theorem [K][4]
8. State the theorem which gives the method of solving linear equations of the form $Pp + Qq = R$. [K][4]

Part B

Answer any five questions. Each question carries 6 marks

9. Check whether the differential equation $(2x \sin y + y^3 e^x)dx + (x^2 \cos y + 3y^2 e^x)dy = 0$ is exact or not. [A][1]
10. Solve $\frac{dy}{dx} + (\frac{2x+1}{x})y = e^{-3x}$ [A][1]
11. Solve $(x + 2y + 3)dx + (2x + 4y - 1)dy = 0$ [A][1]
12. Find the orthogonal trajectories of the family of parabolas $y = cx^2$. [A][1]
13. Show that the solutions $\sin x$ and $\cos x$ of $\frac{d^2 y}{dx^2} + y = 0$ are linearly independent. [U][3]

14. Given that $y = x$ is a solution of $(x^2 + 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$. Find a linearly independent solution by reducing the order. [A][3]
15. Find the integral curve of the equations $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$. [A][4]
16. Find the general solution of the differential equation $x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x+y)z$. [A][4]

Part C

**Answer any three questions.
Each question carries 10 marks**

17. Solve the initial value problem $(2x\cos y + 3x^2y)dx + (x^3 - x^2\sin y - y)dy = 0$, $y(0) = 2$. [A][1]
18. (i) Find the curve for which the normal makes equal angles with the radius vector and the initial line. [A][2]
(ii) Find the shape of a reflector such that light coming from a fixed source is reflected in parallel rays. [A][2]
19. Solve the following differential equation using a special transformation $(x - 2y + 1)dx + (4y - 3y - 6)dy = 0$ [A][1]
20. Solve $(x^2 + 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 6(x^2 + 1)^2$. [A][3]
21. Solve the equation $\frac{dx}{y+\alpha z} = \frac{dy}{z+\beta x} = \frac{dz}{x+\gamma y}$ [A][4]
22. (i) If u is a function of x, y and z which satisfies the PDE $(y - z)\frac{\partial u}{\partial x} + (z - x)\frac{\partial u}{\partial y} + (x - y)\frac{\partial u}{\partial z} = 0$. Show that u contains x, y and z only in combinations $x + y + z$ and $x^2 + y^2 + z^2$. [A][4]
(ii) Find the general integrals of the linear PDE $z(xp - yq) = y^2 - x^2$ [A][4]

MAHATMA GANDHI UNIVERSITY, KOTTAYAM

MGU-UGP (Honours)

FIFTH SEMESTER EXAMINATION

(2024 Admission Onwards)

MG5DSEMAT301: MATHEMATICAL MUSINGS BEYOND CLASSROOM

Duration: 2 Hours

Maximum Marks: 70

(Students should attempt atleast one question from each course outcome to enhance their overall outcome attainability)

Part A

Answer any five questions. Each question carries 2 marks.

1. How did the Indian Mathematician Brahmagupta define zero as a number and placed in the number line? [U][2]
2. What mathematical challenge did Paul Erdos pose to the little boy Louis Posa over lunch and how did Posa solve it? [K][5]
3. What explanation did Ramanujan give to G.H. Hardy about his mathematical formulas? [U][3]
4. Mention the three African American women mathematicians whose real life contributions at NASA are depicted in the film “Hidden Figures”. [K][4]
5. Mention any two major texts (and the respective authors) that constitute the primary evidence of the work of Kerala School of Mathematics. [K][2]
6. Write the Madhava-Gregory Series for the inverse tangent and deduce the Liebnitz series for $\frac{\pi}{4}$. [K][1]
7. How does the humanist philosophy distinguish between “Counting Numbers” and “Pure Numbers”? [An] [7]
8. State Cantor’s theorem. [K][1]

Part B

Answer any five questions. Each question carries 2 marks.

9. Describe the mathematical consequences of performing the basic operations (addition, multiplication and division) with zero. [U][2]
10. Explain the following principle using Galton board: “things that seem hopelessly random and unpredictable when viewed in isolation often turn out to be lawful and predictable when viewed in aggregate”. [A][3]

11. Discuss the ethical dilemma faced by Alan Turing's team immediately after breaking the Enigma code and explain how they resolved it. [E][5]
12. Write a short note on an introduction to mathematics on the silver screen. [An] [4]
13. Explain the Madhava's rule for generating the power series for Indian Sine and Indian Versine. Also write the series. [U][2]
14. Analyze the navigational and calendrical motivations that drove European interest in Indian scientific knowledge during the 16th and 17th centuries. [An] [6]
15. Discuss how the mathematical understanding of the number "2" has changed and evolved over time. [An] [5]
16. Compare formalism, logicism and intuitionism. [An] [7]

Part C

Answer any three questions. Each question carries 10 marks.

17. Explain Page Rank and the Page Rank Algorithm using examples. [A][6]
18. Write about the mathematical legacy of John Nash presented in the film "A Beautiful Mind". [U][2]
19. Based on the film "The man who knew infinity", describe the early life of Srinivasa Ramanujan in India and discuss the challenges, cultural barriers and health struggles, he faced while living in England. [An] [4]
20. Discuss the socio-cultural background and institutional environment that enabled the development of the Kerala School of Mathematics. [An] [7]
21. "Whether Mathematics is created or discovered"-Critically evaluate the Platonists, formalists and humanist views using examples. [An] [5]
22. Explain Paul Benacerraf's under determination problem regarding the reduction of natural numbers to set-theoretic constructions. [An] [5]

MAHATMA GANDHI UNIVERSITY , KOTTAYAM
MGU-UGP (Honours)
Fifth Semester Examination
(2024 Admission Onwards)

MG5DSEMAT302 AN INVITATION TO FUZZY MATHEMATICS
Duration : 2 hours **Maximum Marks :70**

(Students should attempt at least one question from each course outcome to enhance their overall attainability)

Part A

Answer any Five questions Each question carries 2 marks

1. Define Cartesian product of two sets. [U][1]
2. Define extension principle for Fuzzy sets. [R][1]
3. Write about the axiomatic skeleton for fuzzy intersections. [U][2]
4. Write Formula for finding new t - norm if a t - norm is given. [R][2]
5. Define Reflexive , Ir reflexive and Anti - reflexive relations [U][3].
6. Define α – Compatibility class. [R][3]
7. Define a connected Graph. [U][4]
8. Let $V = \{ a, b, c \}$. Define Fuzzy sets σ on V and μ on $E = \{ ab, bc, ac \}$ so that $G = (V, \sigma, \mu)$ is a Fuzzy Graph. [A][4]

Part B

Answer any five Questions, Each question carries 6 marks

9. Prove that a fuzzy set A on R is convex iff $A(\lambda x_1 + (1 - \lambda) x_2) \geq \min \{A(x_1), A(x_2)\}$ for all $x_1, x_2 \in R$ and all $\lambda \in [0,1]$, where $\min$ denotes the minimum operator [A][1]
10. Is standard Fuzzy complement cutworthy ? Justify [An][1]
11. Define equilibrium point in Fuzzy complement and prove that every fuzzy complement has atmost one equilibrium [A][2]
12. Write pseudo - inverse of decreasing generator and find pseudo - inverse for $f(a) = 1 - a^p$, for any $a \in [0,1]$, $p > 0$ [A][2]
13. Show that the relation $R(X, X) = \{ (x, y) : x \text{ and } y \text{ have same remainder when divided by } 3 \}$ is an equivalence relation. [A][3]
14. Explain domain , range and height of a fuzzy relation $R(X, Y)$ with equations. [U][3]
15. Let $G = (V, \sigma, \mu)$ be a Fuzzy graph . Define the following.
 - a) Spanning Fuzzy subgraph of G
 - b) Threshold graph of G corresponding to t , where $0 \leq t \leq 1$ [A][4]

16. Characterize Fuzzy cut vertices and Fuzzy bridges of Fuzzy graphs using maximum spanning trees [A][4]

Part C

Answer any Three Questions, Each question carries 10 marks

17. State and Prove second decomposition theorem in Fuzzy sets. [A][1]

18. a) Prove that equilibrium of any fuzzy complement is its own dual point.

- b) Prove that for each $a \in [0,1]$, $d_a = c(a)$ if and only if $c(c(a)) = a$ [A][2]

19. Write $u_{\max}(a,b)$ and prove that $\max(a,b) \leq u(a,b) \leq u_{\max}(a,b)$ for all $a,b \in [0,1]$ [A][2]

20. a) Define crisp relation and characteristic function

- b) Define Fuzzy relation

- c) Let R be a relation among the three sets $X = \{\text{English, French}\}$

$$Y = \{\text{Dollar, Pound, Franc, Mark}\}$$

$$Z = \{\text{US, France, Britain, Germany}\}$$

Such that $R = \{(\text{English, Dollar, US}), (\text{French, Franc, France}), (\text{English, Dollar, Canada}), (\text{French, Dollar, Canada}), (\text{English, Pound, Britain})\}$. Represent this Relation

using 3 - dimensional membership array. [An][3]

21. Let $G = (\sigma, \mu)$ be a Fuzzy graph. Prove that xy is a Fuzzy bridge if and only if xy is not the weakest edge of any cycle. [A][4]

22. If $G = (\sigma, \mu)$ is a complete Fuzzy graph, Prove that $\mu^\infty = \mu$.

Is the converse true? Justify. [An][4]

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MAHATMA GANDHI UNIVERSITY, KOTTAYAM
MGU-UGP (HONOURS)
FIFTH SEMESTER
B.Sc(Hons) DEGREE EXAMINATION MATHEMATICS
(2024 ADMISION ONWARDS)
MG5DSEMAT303–EXPLORING THE HARMONY OF AUTOMATA

Duration: 2 hrs

Maximum Marks: 70

Students should attempt at least one question from each course outcome to enhance their overall outcome attainability.

Part A

Answer any five questions. Each question carries 2 marks

1. Define a set. What is meant by the elements (members) of a set?[K][1].
2. Define a k -tuple and give an example of a 3-tuple. [U][1].
3. Give the formal definition of a finite automaton as a 5-tuple, naming each component. [K][2]
4. When does a nondeterministic finite automaton accept an input string? [U][2]
5. Define the language of a grammar, and define a context-free language. [K][3]
6. What is the role of the stack in a pushdown automaton? [U][3]
7. What are the three possible outcomes when a Turing machine is run on an input? [K][4]
8. Define a multitape Turing machine and write down its transition function. [U][4]

Part B

Answer any five questions. Each question carries 6 marks

9. What is a binary relation? Give an example of a binary relation on $\{1, 2, 3\}$. [U][1]
10. Define paths, simple paths, cycles, simple cycles, connected graphs and trees, illustrating each with a diagram. [U][1]
11. Prove that the class of regular languages is closed under the operation of concatenation, using a nondeterministic finite automaton in the construction. [A][2]
12. Give the formal definition of a regular expression and of the language $L(R)$ that it describes, explaining each of the six clauses of the inductive definition with an example. [U][2]
13. Prove the lemma that if a language is context free then some pushdown automaton recognizes it, describing carefully the construction of the PDA from a given context-free grammar. [A][3]
14. State and prove the pumping lemma for context-free languages. [An][3]
15. Define a nondeterministic Turing machine and describe its computation as a tree of configurations, explaining when the machine accepts and when it rejects. [U][4]
16. Prove that if an enumerator E enumerates a language A , then some Turing machine M recognizes A . [A][4]

Part C

Answer any three questions. Each question carries 10 marks

17. (a) Define the values TRUE and FALSE and the operations $\neg$ (NOT), $\wedge$ (AND), $\vee$ (OR), $\oplus$ (XOR), $\rightarrow$ (IMPLIES) and $\leftrightarrow$ (EQUALITY) by truth tables.
(b) Verify the distributive law $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$ and both De Morgan laws using truth tables. [U][1]
18. Prove that a language is regular if and only if some regular expression describes it. [An][2].

19. Prove that none of the following languages is regular. In each case exhibit the string chosen and examine every possible decomposition:

- (a) $\{0^n 1^n : n \geq 0\}$;
- (b) $\{ww : w \in \{0, 1\}^*\}$;
- (c) $\{1^{n^2} : n \geq 0\}$.

[An][2]

20. Prove that neither of the following languages is context free

- (a) $C = \{a^i b^j c^k : 0 \leq i \leq j \leq k\}$;
- (b) $D = \{ww : w \in \{0, 1\}^*\}$.

[An][3]

21.

- (a) Define string order on Σ^* .
- (b) Prove that a language is decidable if and only if some enumerator enumerates it in string order.
- (c) Explain why the corresponding statement without the ordering requirement characterises only the Turing-recognizable languages.

[An][4]

22. State the Church–Turing thesis and discuss it in detail.

[U][4]

MAHATMA GANDHI UNIVERSITY KOTTAYAM

MGU- UGP (HONOURS) REGULAR EXAMINATION

FIFTH SEMESTER

**Skill Enhancement Course (SEC) - MG5SECMAT300
Introduction to Python for Mathematical Computation**

2024 Admission Onwards

Maximum Time:-75 Minutes

Maximum Marks:-50

Part A

(Short Answer Type Questions)

Answer any 5 questions

Each question carries 2 marks

1. Determine the data type of the variable x after each assignment:

$$x = 10, \quad x = 10.5, \quad x = 3 + 4j, \quad x = \text{'Python'}.$$

[U] / [CO1]

2. Consider the following code:

```
x = 1
while x <= 6:
    if x == 3:
        x = x + 1
        continue
    if x == 5:
        break
    print(x)
    x = x + 1
```

What will be the output? Explain the role of `continue` and `break` in the program. [U] / [CO1]

3. Write a Python program using a `for` loop and an `if` statement to print all even numbers from 1 to 20 [U] / [CO1]

4. What is NumPy? State any two advantages of using NumPy arrays over Python lists. [A] / [CO3]

5. What is the purpose of the `arange()` function? Write the output of:

```
arange(2.0, 3.0, 0.2)
```

[A] / [CO3]

6. Define the dot product of two vectors. Using

$$A = [1, 2, 3], \quad B = [4, 5, 6],$$

find their dot product.

[A] / [CO3]

-
7. What is meant by a subplot in Matplotlib? What do the three arguments in `subplot(2,1,1)` represent? [A] / [CO4]
8. Define a polar plot. Write the Matplotlib statement used to plot a point using polar coordinates (θ, r) . [A] / [CO4]

Part B
(Short Essay Questions)
Answer any 4 questions
Each question carries 5 marks

9. Write a Python program to read numbers from the keyboard until their sum exceeds 200. Modify the program so that numbers greater than 99 are ignored. Explain the working of the program. [U] / [CO1]
10. Explain how a Python list can be converted into a NumPy array. Write a Python program to create the following two-dimensional array:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

and display the array.

[A] / [CO3]

11. Explain the working of the `arange()` and `linspace()` functions with their syntax. Write Python statements to generate:
- (a) an array from 2 to 3 with a step size of 0.2,
 - (b) an array of 11 equally spaced values from 1 to 2.

[A] / [CO3]

12. What is a vectorized function? Explain how vectorized functions eliminate the need for explicit loops. Using NumPy, evaluate

$$\log_{10}(x)$$

for

$$x = [1, 10, 100, 1000].$$

[A] / [CO3]

13. Write a Python program to plot a circle of radius $a = 5$ using parametric equations

$$x = a \cos \theta, \quad y = a \sin \theta,$$

where $0 \leq \theta \leq 2\pi$. Label the axes and display the resulting curve.

[A] / [CO4]

14. Write a Python program using `meshgrid()` to generate the surface

$$z = \sin(x) + \sin(y),$$

for $-3\pi \leq x, y \leq 3\pi$. Use a suitable 3D plotting method to display the surface and explain the role of the `meshgrid`. [S] / [CO4]

Part C
(Essay Questions)
Answer any 2 questions
Each question carries 10 marks

15. Write a Python program to process the following list:

$$L = [123, 12.4, \text{'haha'}, 3.4].$$

- (a) Use a `for` loop to display all the elements of the list.
- (b) Insert a new element after 12.4 and append another element to the list.
- (c) Explain the difference between inserting and appending an element to a Python list.

[A] / [CO2]

16. Write a Python program using NumPy to create a one-dimensional array containing the elements

$$5, 10, 15, 20, 25.$$

Using the same array, perform the following operations:

- (a) Display the array.
- (b) Extract the first three elements using slicing.
- (c) Find the number of elements in the array.
- (d) Create a new array containing the elements multiplied by 2.

[A] / [CO3]

17. Given the two vectors

$$A = [1, 2, 3], \quad B = [4, 5, 6],$$

write a Python program using NumPy to:

- (a) Calculate the dot and cross product of A and B .
- (b) Display the results.
- (c) Explain the difference between dot product and cross product.

[A] / [CO3]

18. Write a Python program using Matplotlib to plot the function

$$z = \sin(x) + \sin(y)$$

as a three-dimensional surface for

$$-3\pi \leq x, y \leq 3\pi.$$

Use `meshgrid()` to generate the coordinate matrices and `plot_surface()` to display the surface. Explain the steps involved and describe the role of the meshgrid in the program. [S] / [CO4]