



QP CODE: 26151746

Reg No :

Name :

MSc DEGREE (CSS)PRIVATE EXAMINATION, MAY 2026

First Semester

M Sc MATHEMATICS

Core Courses - ME010105 - GRAPH THEORY

2019 Admission Onwards

44D87289

Time: 3 Hours

Weightage: 30

Instructions: (Applicable for 2020 admission onwards): This question Paper contains **two sections**. Answer **section I** questions in the answer book provided. **Section II internal examination questions** must be answered in the **question paper itself**. Follow the detailed instructions given under **section II**.

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define the complement of a graph and self complementary graph. Give an example of a selfcomplementary graph of size 3
2. If the bipartite graph $G(X, Y)$ is regular, show that $|X| = |Y|$
3. An edge $e = xy$ of a connected graph G is a cut edge of G if and only if e belongs to no cycle of G . Prove.
4. Give an example of
 - (i) A tree with just one central vertex that is also a centroidal vertex;
 - (ii) A tree with 2 centroidal vertices, one of which is also a central vertex;
 - (iii) A tree with a disjoint centroid and centre;
5. Write a short note on any two particular cases of the connector problem.
6. Define Eulerian graph with an example.
7. For any vertex v of the Petersen graph P , prove that $P-v$ is Hamiltonian.
8. For a simple graph G , prove that $\chi(G) \geq \alpha(G)$
9. *What you mean by the faces of a planar graph.*
10. *What is the spectrum of K_n*

(8×1=8 weightage)





Part B (Short Essay/Problems)

Answer any **six** questions.

Weight **2** each.

11. Find the order and size of $G_1 \vee G_2$
12. Show that every vertex of a disconnected tournament T on n vertices with $n \geq 3$ is contained in a directed k -cycle, $3 \leq k \leq n$.
13. State and prove Ear decomposition theorem of a block.
14. If e is a loop of a connected graph G , then prove $\tau(G) = \tau(G - e) + \tau(G \circ e)$
15. Find the chromatic number of
 - (a) Petersen graph
 - (b) Wheel W_n
 - (c) An odd cycle with a pendant edge
 - (d) A bipartite graph
 - (e) The Herschel graph
16. Prove that in a critical graph G , no vertex cut is a clique.
17. State and prove Euler formula for a connected plane graph.
18. Define dual of a plane graph. Draw dual of Herschel graph.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight **5** each.

19.
 - a) Define line graph and show that a simple connected graph G is isomorphic to its line graph if and only if it is a cycle.
 - b) For a simple graph G prove that $m(L(G)) = \frac{1}{2} \sum_{i=1}^n d_i^2 - m$
20.
 - a. Prove for any loopless connected graph G , $\kappa(G) \leq \lambda(G) \leq \delta(G)$.
 - b. Show that connectivity and edge connectivity of a simple cubic graph G are equal.
21.
 - (a) Define closure of a graph. Prove that a graph has one and only one closure.
 - (b) Prove that if $cl(G)$ is hamiltonian then G is hamiltonian.
 - (c) Give an example of a graph G such that $cl(G)=G$
22. Prove that every planar graph is 5 – vertex colorable.

(2×5=10 weightage)

