

QP CODE: 26151745



Reg No :

Name :

M.Sc DEGREE (CSS) PRIVATE EXAMINATION, MAY 2026

First Semester

M Sc MATHEMATICS

Core Courses - ME010104 - REAL ANALYSIS

2019 Admission Onwards

7D447E14

Time: 3 Hours

Weightage: 30

Instructions: (Applicable for 2020 admission onwards): This question Paper contains two **sections**. Answer **section I** questions in the answer book provided. **Section II internal examination questions** must be answered in the **question paper itself**. Follow the detailed instructions given under **section II**.

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight **1** each.

1. Define total variation. Prove that $0 \leq V_f < \infty$ and $V_f = 0$ if and only if f is a constant on $[a, b]$.
2. Let f be continuous on $[a, b]$. Then prove that f is of bounded variation on $[a, b]$ if f can be expressed as the difference of two increasing continuous functions.
3. Define upper and lower Riemann Stieltjes sum for a real bounded function.
4. If $|f(x)|$ is Riemann integrable then show that $f(x)$ need not be Riemann integrable.
5. If $f \in \mathcal{R}(\alpha)$ on $[a, b]$ then show that $|\int_a^b f d\alpha| \leq \int_a^b |f| d\alpha$.
6. Show that $\lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} S_{m,n} \neq \lim_{n \rightarrow \infty} \lim_{m \rightarrow \infty} S_{m,n}$, if $S_{m,n} = \frac{m}{m+n}$, $(m, n = 1, 2, 3, \dots)$.
7. If the sequence of functions $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E , prove that $\{f_n + g_n\}$ converges uniformly on E .
8. Is every Cauchy sequence convergent? If no, when will it be convergent?
9. State a theorem concerning an inversion in the order of summation of a double sequence.
10. Define the exponential function $E(z)$ and prove that $E(0) = 1$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any **six** questions.

Weight **2** each.

11. If f is continuous on $[a, b]$ and if f' exists and is bounded in the interior, with $|f'(x)| \leq A$ for all x in





- (a, b) , then f is of bounded variation on $[a, b]$. Illustrate the validity of this statement using an example.
12. Prove using an example that there exists functions with same graph which are not equivalent.
 13. If f is monotonic on $[a, b]$ and if α is continuous on $[a, b]$ then prove that $f \in \mathcal{R}(\alpha)$.
 14. Suppose φ is a strictly increasing continuous function that maps an interval $[A, B]$ onto $[a, b]$. Suppose α is monotonically increasing on $[a, b]$ and $f \in \mathcal{R}(\alpha)$ on $[a, b]$. Define β and g on $[A, B]$ by $\beta(y) = \alpha(\varphi(y))$, $g(y) = f(\varphi(y))$. Then prove that $g \in \mathcal{R}(\beta)$ and $\int_A^B g d\beta = \int_a^b f d\alpha$.
 15. Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathcal{R}(\alpha)$ on $[a, b]$, for $n = 1, 2, 3, \dots$ and suppose $f_n \rightarrow f$ uniformly on $[a, b]$. Then prove that $f \in \mathcal{R}(\alpha)$ on $[a, b]$.
 16. Obtain a series from $\phi(x) = |x|$, $(-1 \leq x \leq 1)$ and $\phi(x+2) = \phi(x)$ for all real x , which converges uniformly on $\mathbb{R}^1$.
 17. Prove by an example that for a uniformly bounded sequence of continuous functions, there need not exist a pointwise convergent subsequence, even if the domain is compact.
 18. If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E , prove that $\{f_n\}$ has a subsequence which converges for every $x \in E$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight **5** each.

19. (i) Let $f : [a, b] \rightarrow \mathbb{R}^n$. Define the terms graph, path and motion for f . Also using an example, show that different paths can trace out the same curve.
 (ii) Define rectifiable paths. Give an example of a non-rectifiable curve.
 (iii) Let $f : [a, b] \rightarrow \mathbb{R}^n$ with components $f = (f_1, f_2, \dots, f_n)$. Then prove that f is rectifiable if and only if each of the components f_k is of bounded variation on $[a, b]$. Also prove that if f is rectifiable then we have the inequalities $V_k(a, b) \leq \Lambda_f(a, b) \leq V_1(a, b) + \dots + V_n(a, b)$, $k = 1, 2, \dots, n$, where $V_k(a, b)$ denotes the total variation of f_k on $[a, b]$.
20. Suppose F and G are differentiable functions on $[a, b]$, $F' = f \in \mathcal{R}$ and $G' = g \in \mathcal{R}$. Then prove that $\int_a^b F(x)g(x)dx = F(b)G(b) - F(a)G(a) - \int_a^b f(x)G(x)dx$.
21. If $\{f_n\}$ is a sequence of continuous functions on E , and if $f_n \rightarrow f$ uniformly on E , then prove that f is continuous on E .
22. Prove that, for every interval $[-a, a]$ there exists a sequence of real polynomials P_n such that $P_n(0) = 0$ and such that $\lim_{n \rightarrow \infty} P_n(x) = |x|$ uniformly on $[-a, a]$.

(2×5=10 weightage)

