



QP CODE: 26151743



26151743

Reg No :

Name :

M.Sc DEGREE (CSS) PRIVATE EXAMINATION, MAY 2026

First Semester

M Sc MATHEMATICS

Core Courses - ME010102 - LINEAR ALGEBRA

2019 Admission Onwards

8FF07C63

Time: 3 Hours

Weightage: 30

Instructions: (Applicable for 2020 admission onwards): This question Paper contains **two sections**. Answer **section I** questions in the answer book provided. **Section II internal examination questions** must be answered in the **question paper itself**. Follow the detailed instructions given under section II.

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight **1** each.

1. Let V be the set of pairs (x, y) of real numbers and let F be the field of real numbers. Define $(x, y) + (x_1, y_1) = (x + x_1, 0)$ and $c(x, y) = (cx, 0)$. Is V , with these operations, a vector space over F ?
2. Are the vectors $\alpha_1 = (1, 1, 2, 4)$, $\alpha_2 = (2, -1, -5, 2)$, $\alpha_3 = (1, -1, -4, 0)$, $\alpha_4 = (2, 1, 1, 6)$ linearly independent in $\mathbb{R}^4$?
3. If V is a vector space over the field F and U, T_1 and T_2 are linear operators on V , then prove that $U(T_1 + T_2) = UT_1 + UT_2$ and $c(UT_1) = (cU)T_1$ where c is a scalar.
4. Show that the space C of complex numbers and $\mathbb{R}^2$ are isomorphic, considering as vector spaces over $\mathbb{R}$.
5. If $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear operator defined as $T(x_1, x_2) = (-x_2, x_1)$, find the matrix of T in the standard ordered basis for $\mathbb{R}^2$.
6. Show that the function D from $K^{n \times n}$ to K defined by $D(A) = a A(1, k_1) \dots A(n, k_n)$ is n -linear, where $a \in K$ and $k_1, \dots, k_n$ are positive integers less than or equal to n .
7. Let D be a 2-linear function with the property that $D(A) = 0$ for all 2×2 matrices A over K having equal rows. Then show that D is alternating.
8. Show that determinant of an invertible matrix is non-zero.
9. Prove that similar matrices have the same characteristic values
10. Let W be an invariant subspace for T . Prove that the characteristic polynomial of the restriction operator T_W divides the characteristic polynomial of T .

(8×1=8 weightage)





Part B (Short Essay/Problems)

Answer any **six** questions.

Weight **2** each.

11. Show that the intersection of any collection of subspaces of a vector space V is a subspace of V .
12. Show that row-equivalent matrices have same row space.
13. Describe the range and null space of the differential operator on the vector space V of all polynomial functions over a field F . Hence determine its rank and nullity
14. If S is any subset of a finite dimensional vector space V and W is the subspace of V spanned by S then prove that $S^{00} = W$
15. Let V and W be finite dimensional vector spaces over the field F and $T : V \rightarrow W$ is a linear transformation. Prove that $\text{Rank}(T^t) = \text{Rank } T$.
16. Show that $\det(AB) = (\det A)(\det B)$, if A and B are two $n \times n$ matrices over K .
17. Let T be a linear operator on $\mathbb{R}^3$ which is represented in the standard ordered basis by the matrix

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$
 . Prove that T is diagonalizable by exhibiting a basis for $\mathbb{R}^3$, each vector of which is a characteristic vector of T
18. Let $V = W_1 \oplus \cdots \oplus W_k$, prove that there exist k linear operators $E_1, E_2, \dots, E_k$ on V such that
 1. Each E_i is a projection
 2. $E_i E_j = 0$ if $i \neq j$
 3. $I = E_1 + \cdots + E_k$
 4. The range of E_i is W_i

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight **5** each.

19. (a) Let V be an n -dimensional vector space over the field F , and let $\mathcal{B}$ and $\mathcal{B}'$ be two ordered bases of V . Then show that there is a unique, necessarily invertible, $n \times n$ matrix P with entries in F such that (i) $[\alpha]_{\mathcal{B}} = P[\alpha]_{\mathcal{B}'}$ and (ii) $[\alpha]_{\mathcal{B}'} = P^{-1}[\alpha]_{\mathcal{B}}$ for every vector α in V .
 (b) Let $\mathcal{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ be the ordered basis for $\mathbb{R}^3$ consisting of $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 1, 1)$, $\alpha_3 = (1, 0, 0)$. What are the coordinates of the vector (a, b, c) in the ordered basis $\mathcal{B}$?
20. Let V be a finite dimensional vector space over the field F , then prove that
 - (i) $\dim V = \dim V^*$ by exhibiting a unique dual basis $\mathcal{B}^* = \{f_1, f_2, f_3, \dots, f_n\}$ for any ordered basis





$B = \{\alpha_1, \alpha_2, \alpha_3 \dots \alpha_n\}$ of V and

(ii) $f = \sum_{i=1}^n f(\alpha_i) f_i$ and $\alpha = \sum_{i=1}^n f_i(\alpha) \alpha_i$ where f is any linear functional and α is any vector in V .

21. (a) Let A be an $n \times n$ matrix over K . Show that A is invertible over K if and only if $\det A$ is invertible in K and that if A is invertible, the unique inverse for A is $A^{-1} = (\det A)^{-1} \text{adj } A$.

(b) Use Cramer's rule to solve the following system of linear equations over the field of rational numbers:

$$-x + 3y - 2z = 5; 4x - y - 3z = -8; 2x + 2y - 5z = 7.$$

22. Let T be a linear operator on a finite dimensional vector space V . If f is the characteristic polynomial for T , then prove that $f(T) = 0$

(2×5=10 weightage)

