



26151742

QP CODE: 26151742

Reg No :

Name :

MSc DEGREE (CSS) PRIVATE EXAMINATION, MAY 2026

First Semester

M Sc MATHEMATICS

Core Courses - ME010101 - ABSTRACT ALGEBRA

2019 Admission Onwards

Time: 3 Hours

Weightage: 30

Instructions: (Applicable for 2020 admission onwards): This question Paper contains **two sections**. Answer **section I** questions in the answer book provided. **Section II internal examination questions** must be answered in the **question paper itself**. Follow the detailed instructions given under **section II**.

Part A (Short Answer Questions)Answer any **eight** questions.Weight **1** each.

1. Find all abelian groups upto isomorphism of order 1089.
2. Let H be a subgroup of a group G . Prove that G is an H -set under the action $*(h, g) = hg$ for $h \in H$ and $g \in G$.
3. Define torsion subgroup of an abelian group. Find the torsion subgroup of $\mathbb{Z}_4 \times \mathbb{Z} \times \mathbb{Z}_3$
4. Find the kernel of the homomorphism $\phi: \mathbb{Z}_{18} \rightarrow \mathbb{Z}_{12}$, where $\phi(1) = 10$.
5. Let G be a finite group and let P be a normal p -subgroup of G . Show that P is contained in every Sylow p -subgroup of G .
6. Let G be a finite group and let primes p and $q \neq p$ divide $|G|$. Prove that if G has precisely one proper Sylow p -subgroup, then G is not simple.
7. Show that $2^{11,213} - 1$ is not divisible by 11
8. Find the quotient and remainder when $f(x) = x^4 - 3x^3 + 2x^2 + 4x - 1$ is divided by $g(x) = x^2 - 2x + 3$ in $\mathbb{Z}_5[x]$
9. Define Quaternion ring.
10. Define ideal of a ring. Give an example.

(8×1=8 weightage)





Part B (Short Essay/Problems)

Answer any **six** questions.

Weight **2** each.

11. Let H be a normal subgroup of a group G . Prove that the cosets of H form a group G/H under the binary operation $(aH)(bH) = (ab)H$.
12. Let X be a G -set and let $x \in X$. Prove that $|Gx| = (G : G_x)$.
13. Let H be a p -subgroup of a finite group G . Prove that $(N[H] : H) \equiv (G : H) \pmod{p}$.
14. Prove that the center of a finite nontrivial p -group G is nontrivial.
15. Show that for $[(a,b)]$ and $[(c,d)]$ in F the equations $[(a,b)] + [(c,d)] = [(ad+bc, bd)]$ and $[(a,b)] [(c,d)] = [(ac, bd)]$ give well defined addition and multiplication on F where F be the desired field of quotients of an integral domain D .
16. How many polynomials are there of degree ≤ 3 in $\mathbb{Z}_2[x]$? Find all irreducible polynomials of degree 2 in $\mathbb{Z}_2[x]$.
17. Explain endomorphism of rings and ring of endomorphisms.
18. Let $p(x)$ be an irreducible polynomial in $F[x]$. If $p(x)$ divides $r(x)s(x)$ for $r(x), s(x) \in F[x]$, then prove that either $p(x)$ divides $r(x)$ or $p(x)$ divides $s(x)$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight **5** each.

19. (a) Prove that a direct product of a finite number of abelian groups forms an abelian group.
(b) Prove that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic and isomorphic to $\mathbb{Z}_{mn}$ if and only if m and n are relatively prime.
20. (a) If H and K are finite subgroups of a group G , then prove that $|HK| = \frac{(|H|)(|K|)}{|H \cap K|}$.
(b) Prove that no group of order 48 is simple.
21. Prove that $R[x]$ of all polynomials with coefficients in a ring R is a ring under polynomial addition and polynomial multiplication. Also show that if R is commutative, then $R[x]$ is commutative.





22. (a) Prove that if R is a ring with unity and N is an ideal of R containing a unit, then $N = R$.
(b) Prove that a field contains no proper non trivial ideals.
(c) Let R be a commutative ring with unity. Define maximal ideal and factor ring of R . Then prove that M is maximal ideal of R if and only if R/M is a field.

(2×5=10 weightage)

