

26023814



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Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2026

Fourth Semester

Faculty of Science

Branch I (A)—Mathematics

MT04E13—ALGORITHMIC GRAPH THEORY

(2016—2018 Admissions—Mercy Chance)

Time : Three Hours

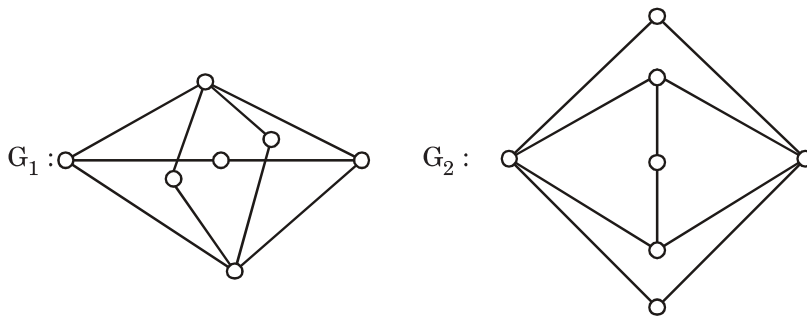
Maximum Weight : 30

Part A

Answer all question.

Each question carries 1 weight.

1. A graph $G = (V, E)$ of order 8 has the power set of the set $S = \{1, 2, 3\}$ as its vertex set. Draw the graph G , determine the degree G , determine the degree of each vertex of G and determine the size of G .
2. Determine whether $G_1 \cong G_2$ where



3. Give an example of a tree of order 8 containing six vertices of degree 1 and two vertices of degree 4.
4. Define the terms diameter, radius and center of a connected graph G .

Turn over





26023814

5. Define the terms maximum flow and minimum cut in a network.
6. Define the terms vertex-connectivity and edge connectivity.
7. Show that for each positive integer k , the complete graph K_{2k} is 1-factorable (One factorable).
8. Prove that every r -regular bipartite graph ($r \geq 1$) has a perfect matching.

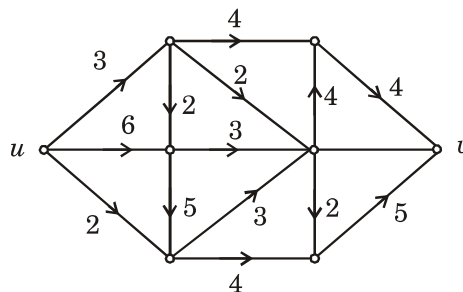
(5 × 1 = 5)

Part B

Answer any **five** questions.

Each question carries 2 weight.

9. Prove that the size of every bipartite graph of order n is at most $\lfloor n^2/4 \rfloor$.
10. Determine whether the sequence 4, 4, 3, 2, 1 is graphical. If so, construct a graph with the appropriate degree sequence.
11. Prove that a non-trivial graph G is a bipartite graph if and only if G contains no odd cycles.
12. Prove that every graph is the center of some graph.
13. State and prove max-flow min-cut theorem.
14. Let N be the network with sources u and sink v given below, where each are is labeled with its capacity.
 - (a) Show that no flow can have a value exceeding 9.
 - (b) Give an example of a flow f on N such that $\text{val}(f) = 9$.





26023814

15. State and prove Hall's Theorem.
16. Prove that every r -regular bipartite graph ($r \geq 1$) has a 1-factor.

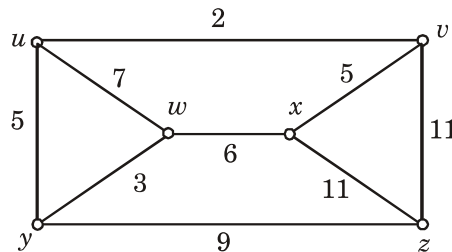
(5 × 2 = 10)

Part C

Answer any **three** questions.

Each question carries 5 weight.

17. (a) Give a description of the Search algorithm and algorithmic complexity.
(b) Determine all non-isomorphic graphs of order 5.
(c) Prove that if G is a graph of order $n \geq 2$ and size $m \geq \left(\frac{n-1}{2}\right) + 1$, then G is connected.
18. (a) Prove that the center of every connected graph of G lies in a single block of G .
(b) Prove that an edge e in a graph G is a bridge of G if and only if e lies on no cycle in G .
(c) Prove that a graph G is a tree if and only if every two vertices of G are connected by a unique path.
19. (a) Prove that if T is a tree of order n and size m , then $m = n - 1$.
(b) Let G be a graph of order n and size m . Prove that if G has no cycles and $m = n - 1$, then G is a tree.
(c) Apply Prim's algorithm to find the minimum spanning tree in the weighted graph given below :



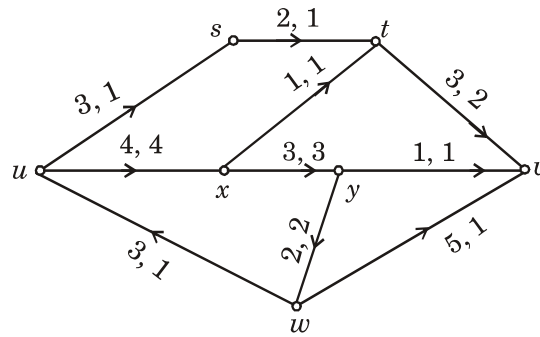
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26023814

20. Establish the arc form of Menger's Theorem.
21. Use Edmonds-Karp algorithm to find a maximum flow f and a minimum cut k in the following network :



22. (a) Prove that a matching m in a graph G is a maximum matching if and only if G contains no m -augmenting path.
- (b) Show that every tree has at most one perfect matching.

(3 × 5 = 15)

