

26023796



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Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2026

Fourth Semester

Faculty of Science

Branch I (A)—Mathematics

MT04E02—COMBINATORICS

(2016—2018 Admissions—Mercy Chance)

Time : Three Hours

Maximum Weight : 30

Part A

*Answer any **five** questions
Each question carries a weight of 1.*

1. In how many ways can a committee of 5 be formed from a group of 11 people consisting of 4 teachers and 7 students if :
 - (a) there is no restriction in the selection.
 - (b) the committee must include exactly 2 teachers.
2. Find the number of ternary sequences of length 10 having two 0's, three 1's and five 2's.
3. State the pigeonhole principle.
4. Show that given any set of 5 numbers, there are 3 numbers in the set whose sum is divisible by 3.
5. Define Euler's ϕ - function and compute $\phi(100)$.
6. Let A,B and C be finite sets. Show that $|\bar{A} \cap B| = |B| - |A \cap B|$.
7. Define the generating function of a sequence (ar) and find the generating function of $(1, k, k^2, \dots)$, where k is a constant.
8. Find the number of ways to select 4 members from the multiset

$$M = \{2.b, 1.c, 2.d, 1.e\}$$

$$(5 \times 1 = 5)$$

Turn over





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Part B

*Answer any **five** questions*

Each question carries a weight of 2.

9. Let $X = \{1, 2, \dots, 100\}$ and let $S = \{(a, b, c) | a, b, c \in X, a < b \text{ and } a < c\}$.
10. If there must be at least one person in each table, in how many ways can 6 people be seated
 - (i) around two tables.
 - (ii) around three tables.
11. Show that for any set of 10 points chosen within a square whose sides are of length 3 units, there are two points in the set whose distance apart is at most $\sqrt{2}$.
12. Show that among 5 points in an equilateral triangle of unit side length, there are 2 whose distance is at most $\frac{1}{2}$ units apart.
13. Let $S = \{1, 2, \dots, 500\}$. Find the number of integers in S which are divisible by 2, 3 or 5.
14. Let $n \in \mathbb{N}$ and let $n = p_1^{m_1} p_2^{m_2} \dots p_k^{m_k}$ be its prime factorization. Show that

$$\phi(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \dots \left(1 - \frac{1}{p_k}\right).$$
15. (a) In how many ways can 4 of the letters from PAPAYA be arranged.
 (b) Find the co-efficient of x^{20} in the expansion of $(x^3 + x^4 + x^5 \dots)^3$.
16. Solve $a_n - 6a_{n-1} + 9a_{n-2} = 0$ given that $a_0 = 2$ and $a_1 = 3$.

(5 × 2 = 10)





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Part C*Answer any **three** questions.**Each question carries a weight of 5.*

17. (a) Between 2000 and 7000, find the number of even integers in which no digit is repeated.
- (b) Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$, where $n, r \in \mathbb{N}$ with $r \leq n$.
18. (a) Prove that at a gathering of any six people some three of them are either mutual acquaintances or complete strangers to one another.
- (b) Prove that for all integers $p, q \geq 2$ $R(p, q) \leq R(p-1, q) + R(p, q-1)$. Here $R(p, q)$ denote the Ramsey number.
19. (a) Show that $R(p, q) = R(q, p)$ for all $p, q \in \mathbb{N}$.
- (b) Show that $R(2, q) = q$ for all $q \in \mathbb{N}$.
- (c) Show that $R(4, 4) \leq 18$ and $R(3, 6) \leq 19$.
20. (a) Find the number of non-negative integer solutions to the equation $x_1 + x_2 + x_3 = 15$ where $x_1 \leq 5$, $x_2 \leq 6$ and $x_3 \leq 7$.
- (b) State and prove the general principle of inclusion exclusion.
21. (a) Find the number of ways to distribute 10 identical pieces of candy to 3 children so that no child gets more than 4 pieces.
- (b) Find the generating function of the sequence (a_r) where a_r is the number of integer solutions to the equation $x_1 + 2x_2 + 3x_3 + 4x_4 = r$ with $x_i \geq 0$ for each $i = 1, 2, 3, 4$.
22. (a) Solve the recurrence relation.
- $$a_n = a_{n-1} + a_{n-2} \text{ given that } a_0 = 1 \text{ and } a_1 = 1.$$
- (b) Solve the recurrence relation.
- $$a_n - 3a_{n-1} + 2a_{n-2} = 2^n \text{ given that } a_0 = 3 \text{ and } a_1 = 8.$$

(3 × 5 = 15)

