

26023794



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Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2026

Fourth Semester

Faculty of Science

Branch I(A)–Mathematics

MT04E01—ANALYTIC NUMBER THEORY

(2016—2018 Admissions – Mercy Chance)

Time : Three Hours

Maximum Weight : 30

Part A

*Answer any **five** questions.
Each question carries 1 weight.*

1. Define Mobius function $\mu(n)$ and show that for $n \geq 1$ $\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{if } n > 1. \end{cases}$
2. Define the Mangoldt function $\wedge(n)$ and show that for every integer $n \geq 1$ $\log n = \sum_{d|n} \wedge(d)$.
3. Prove that two lattice points (a, b) and (m, n) are mutually visible if and only if $a - m$ and $b - n$ are relatively prime.
4. Define the Chebyshev's functions $\psi(x)$ and $\vartheta(x)$.
5. Show that for $x \geq 2$, $\vartheta(x) = \pi(x) \log x - \int_2^x \frac{\pi(t)}{t} dt$
6. Define the congruence relation and show that $a \equiv b \pmod{m}$ if and only if a and b give the same remainder when divided by m .





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7. Assume $(a, m) = 1$. Show that the linear congruence $ax \equiv b \pmod{m}$ has exactly one solution.
8. Let $(a, m) = 1$ and $f = \exp_m(a)$. Show that $\exp_m(a^k) = \frac{\exp_m(a)}{(k, f)}$.

(5 × 1 = 5)

Part B

*Answer any **five** questions.
Each question carries 2 weight.*

9. Define the Euler's totient function $\phi(n)$ and show that if $n \geq 1$, $\phi(n) = \sum_{d|n} \mu(d) \frac{n}{d}$.
10. Define multiplicative function and show that if f and g are multiplicative, so is their Dirichlet product $f * g$.
11. Define the Liouville's function $\lambda(n)$ and show that for every $n \geq 1$, $\sum_{d|n} \lambda(d) = \begin{cases} 1 & \text{if } n \text{ is a square} \\ 0 & \text{otherwise} \end{cases}$.
12. Prove that for $n \geq 1$ the n^{th} prime p_n satisfies $\frac{1}{6} n \log n < p_n < 12 \left(n \log n + n \log \frac{12}{e} \right)$.
13. State and prove Euler-Fermat theorem.
14. Prove that $5n^3 + 7n^5 \equiv 0 \pmod{12}$ for all integers n .
15. State and prove Lagrange's theorem.
16. Let x be an odd integer. Prove that if $\alpha \geq 3$, $x^{\phi(2^\alpha)/2} \equiv 1 \pmod{2^\alpha}$.

(5 × 2 = 10)





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Part C

Answer any **three** questions.
Each question carries 5 weight.

17. (a) Show that for $n \geq 1$, $\phi(n)n_{p/n}^{\pi} \left(1 - \frac{1}{p}\right)$.

- (b) If f is an arithmetical function with $f(1) \neq 0$, prove that there is a unique arithmetical function f^{-1} , called the Dirichlet inverse of f , such that $f * f^{-1} = f^{-1} * f = I$. Also show that :

$$f^{-1}(1) = \frac{1}{f(1)}, f^{-1}(n) = -\frac{1}{f(1)} \sum_{\substack{d|n \\ d < n}} f\left(\frac{n}{d}\right) f^{-1}(d) \text{ for } n > 1.$$

18. (a) Establish the Euler's summation formula.

- (b) Prove that the set of lattice points visible from the origin has density $6/\pi^2$.

19. Prove that for every integer $n \geq 2$, $\frac{1}{6} \frac{n}{\log n} < \pi(n) < 6 \frac{n}{\log n}$.

20. (a) State and prove Wilson's theorem.

(b) Prove that for any prime $p \geq 5$ $\sum_{k=1}^{p-1} \frac{(p-1)!}{k} \equiv 0 \pmod{p^2}$.

- (c) State and prove Chinese remainder theorem.

21. (a) Prove that set of lattice points in the plane visible from the origin contains arbitrarily large square gaps.

- (b) Solve the congruence $25x \equiv 15 \pmod{120}$.

(c) If p is an odd prime, prove that $1^2 \cdot 3^2 \cdot 5^2 \dots (p-2)^2 \equiv (-1)^{(p+1)/2} \pmod{p}$.

22. State and prove Euler's pentagonal number theorem.

(3 × 5 = 15)

