



QP CODE: 26019093



26019093

Reg No :

Name :

M.Sc DEGREE (CSS) EXAMINATION, MARCH 2026

Fourth Semester

M Sc MATHEMATICS

ELECTIVE - ME810401 - PROBABILITY THEORY

2019 Admission Onwards

62B2B440

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

Weight 1 each.

1. What are the limitations of classical definition of probability.
2. State the principle of inclusion-exclusion.
3. Let (Ω, F, P) be a probability space. Let $B \in F$ with $P(B) > 0$. Then prove that (Ω, F, P_B) where $P_B(A) = P(A/B)$ for all $A \in F$ is a probability space.
4. Show that if X is a random variable, $aX + b$ is also a random variable.
5. Two unbiased dice are thrown. Find the expected value of the sum of the numbers of points on them.
6. If X is a r.v such that $E(X) = 3$ and $E(X^2) = 13$, use Chebycheff's inequality to determine a lower bound for $P(-2 < X < 8)$.
7. State the properties of distribution function of a 2D random variable.
8. Show that $E\left(\frac{X_1 + X_2 + \dots + X_n}{n}\right) = \mu$ if $X_1, X_2, \dots, X_n$ are identically distributed random variables with mean μ .
9. Prove that $X_n \xrightarrow{P} X \Leftrightarrow X_n - X \xrightarrow{P} 0$.
10. Let X_n be a sequence of random variables such that $X_n \xrightarrow{2} X$. Then show that $EX_n \rightarrow EX$.

(8×1=8 weightage)





Part B (Short Essay/Problems)

Answer any **six** questions.

Weight 2 each.

11. Let $\{A_n\}$ be a non increasing sequence of events in S , then show that

$$\lim_{n \rightarrow \infty} P(A_n) = P(\lim_{n \rightarrow \infty} A_n) = P(\cap_{n=1}^{\infty} A_n).$$
12. State and prove multiplication rule of probability in the case of n events $A_1, A_2, \dots, A_n$.
13. Prove that the set of discontinuity points of a distribution function of a r. v X is at the most countable.
14. A symmetric die is thrown 600 times, find the lower bound for the probability of getting 80 to 120 sixes.
15. Let $F(x, y) = \begin{cases} 1 & \text{if } x + 2y > 1 \\ 0 & \text{if } x + 2y < 1 \end{cases}$. Does F define a distribution function in the plane?
16. Find the correlation coefficient between X and Y ; where X and Y are distributed identically with probability mass function

$$P[X = k] = \frac{1}{N}; \quad k = 1, 2, \dots, N.$$
17. Let X_n be a sequence of random variables such that $X_n \xrightarrow{L} X$ and let c be a constant. Then show that
 (i) $X_n + c \xrightarrow{L} X + c$ and (ii) $cX_n \xrightarrow{L} cX$.
18. Examine whether Strong Law of Large numbers holds for the following sequence of independent random variables with $P[X_k = \pm 2^k] = 1/2$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. Five percent of patients suffering from a certain disease are selected to undergo a new treatment that is believed to increase the recovery rate from 30% to 50%. A person is randomly selected from these patients after the completion of the treatment and is found to have recovered. What is the probability that the patient received the new treatment? State the result you used.





20. The random variable X takes the value n with the probability $\frac{1}{2^n}$, $n = 1, 2, 3, \dots$. Find the moment generating function of X and hence find the mean and the variance.
21. Check whether the following function is a joint pdf, $f(x, y) = \begin{cases} x + y & \text{if } 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$.
If so, find $E(X/Y)$ and $Var(Y/X)$.
22. Let $\{X_n\}$ be any sequence of random variables. Let $\bar{Y}_n = \frac{\sum_{i=1}^n X_i}{n}$. Prove that a necessary and sufficient condition for the sequence $\{X_n\}$ to satisfy weak law of large numbers is that
$$E \left\{ \frac{\bar{Y}_n^2}{1 + \bar{Y}_n^2} \right\} \rightarrow 0, \quad \text{as } n \rightarrow \infty$$

(2×5=10 weightage)

