



QP CODE: 26802014



26802014

Reg No :

Name :

INTEGRATED M Sc DEGREE EXAMINATION APRIL 2026

Second Semester

COMPLEMENTARY - ICSC2CM5 - MATHEMATICS – II-LINEAR ALGEBRA

INTEGRATED MSc COMPUTER SCIENCE - ARTIFICIAL INTELLIGENCE & MACHINE
LEARNING, INTEGRATED MSc COMPUTER SCIENCE - DATA SCIENCE

2020 Admission Onwards

9FDB697C

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight 1 each.

1. Define the direct sum of subspaces of a vector space.
2. Write a basis each of a) polynomial functions $\mathcal{P}_m(\mathbb{F})$ b) $\mathbb{F}^n$.
3. Give an example for a linear map.
4. Define matrix addition.
5. If $A = \begin{bmatrix} 8 & 4 & 5 \\ 1 & 9 & 7 \end{bmatrix}$, find $A_{2, \cdot}$ and $A_{\cdot, 2}$.
6. Find the matrix of a vector $x \in \mathbb{F}^n$ with respect to the standard basis.
7. Let $\mathcal{T} \in \mathcal{L}(\mathcal{P}(\mathbb{R}))$ be defined by $\mathcal{T}p = p'$. Is $\mathcal{P}_4(\mathbb{R})$ invariant under $\mathcal{T}$?
8. What is the difference between the upper triangular matrix and a diagonal matrix?
9. State parallelogram equality in vector space.
10. Define orthogonal vectors in a vector space. Give an example of an orthogonal set in $\mathbb{R}^3$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any **six** questions.

Weight 2 each.

11. Show that the following lists of vectors are linearly independent in $\mathbb{F}^3$
a) $(1, 1, 1), (0, -1, 2)$ b) $(0, 1, 1), (0, 2, 1), (1, 5, 3)$.





12. Prove that every spanning list of vectors in a finite dimensional vector space V with length $\dim V$ is a basis of V .
13. Let $\mathcal{T} \in \mathcal{L}(\mathcal{V}, \mathcal{W})$. Then prove that $\mathcal{T}$ is injective if and only if $\text{null } \mathcal{T} = \{0\}$.
14. Prove that two finite-dimensional vector spaces over $\mathbb{F}$ are isomorphic if and only if they have the same dimension.
15. Suppose V is finite dimensional and $\mathcal{S}, \mathcal{T} \in \mathcal{L}(\mathcal{V})$. Prove that $\mathcal{ST}$ and $\mathcal{TS}$ have the same eigenvalues.
16. Let $\mathcal{T} \in \mathcal{L}(V)$ and $(T - 2I)(T - 3I)(T - 4I) = 0$. Suppose λ is an eigenvalue of $\mathcal{T}$. Prove that $\lambda = 2$ or $\lambda = 3$ or $\lambda = 4$.
17. Suppose that V is a real inner product space. Show that the set of self-adjoint operators on V is a subspace of $L(V)$.
18. Suppose $T \in L(V)$ and s is the singular value of T . Prove that there exist a vector $v \in V$ such that $\|v\| = 1$ and $\|T(v)\| = s$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. a) Prove that the span of a list of vectors in V is the smallest subspace of V containing all the vectors in the list.
b) Show that $\mathcal{P}(\mathbb{F})$ is infinite-dimensional.
20. Let V be a finite dimensional vector space and $\mathcal{T} \in \mathcal{L}(\mathcal{V}, \mathcal{W})$. Then prove that $\text{range } \mathcal{T}$ is finite-dimensional and $\dim V = \dim \text{null } \mathcal{T} + \dim \text{range } \mathcal{T}$.
21. Define $\mathcal{T} \in L(\mathbb{F}^3)$ by $\mathcal{T}(x, y, z) = (2x + y, 5y + 3z, 8z)$. Then, find a basis of $\mathbb{F}^3$ with respect to which $\mathcal{T}$ has a diagonal matrix. Also find the diagonal matrix correspond to the new basis.
22. Suppose $T \in L(\mathbb{R}^3)$ has an upper triangular matrix with respect to the basis $(1, 0, 0), (1, 1, 1), (1, 1, 2)$. Find an orthonormal basis of $\mathbb{R}^3$ with respect to which T has an upper triangular matrix.

(2×5=10 weightage)

