



MSc DEGREE (CSS) EXAMINATION, DECEMBER 2025

First Semester

MSC Data Analytics

Core Courses - ST050102 - MATHEMATICAL FOUNDATION FOR DATA ANALYTICS 1

2020 Admission Onwards

8B175D28

Time: 3 Hours

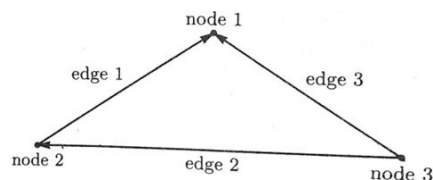
Weightage: 30

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight 1 each.

- Find two points on the line of intersection of the three planes $t = 0$ and $z = 0$ and $x + y + z + t = 1$ in the 4-D space.
- (a) Sketch these 3 lines and decide if the equations are solvable:
 $x + 2y = 2$
 $x - y = 2$
 $y = 1$
 What happens if all right-hand sides are zero? Is there any nonzero choice of right-hand sides that allows the three lines to intersect at the same point?
 (b) Draw the two pictures in two planes for the equations
 $x - 2y = 0$, $x + y = 6$.
- Define (a) subspace & (b) column space.
- For the 3-node triangular graph in the below figure, write the 3 by 3 incidence matrix. Find a solution to $Ax = 0$ and describe all other vectors in the nullspace of A. Find a solution to $A^T y = 0$ and describe all other vectors in the left nullspace of A.



- (i) Define linear transformation.
 (ii) Find whether the following transformation is linear or not.
 $T(x, y) = (x, x)$





6. Find the determinant of the following matrix using the properties of determinant. Also state the properties used for finding it.

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

7. State and prove Cramer's rule.

8. Find the eigen values and eigen vectors of the following matrix

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}.$$

9. Find the saddle points of the following:

a) $f = 2x^2 + 4xy + y^2$

b) $g = 2xy$

10. Decide whether the following matrix is positive definite, negative definite, semi definite or indefinite.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 9 \end{bmatrix}$$

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any **six** questions.

Weight 2 each.

11. Find three matrices E_{21} , E_{31} , & E_{32} . Put A into triangular form U by finding a matrix M such that $MA = U$.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 0 \\ -2 & 2 & 0 \end{bmatrix}$$

12. Find the $PA = LDU$ factorization for the following matrix.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

13. What are the special solutions to $Rx = 0$ and $R^T y = 0$ for the following R's?

$$R = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } R = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

14. Find the dimension of

- (i) The column space of A
- (ii) The column space of U
- (iii) The row space of A
- (iv) The row space of U.



Which two of the spaces are the same?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix} \text{ and } U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

15. Explain determinant with an example

16. Diagonalize the following matrix if possible.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

17. Prove the following.

(i) If $A = A^H$, then for all complex vectors x , the number $x^H Ax$ is real

(ii) If $A = A^H$, every eigen value is real.

(iii) Two eigenvectors of a Hermitian matrix, if they come from different eigenvalues, are orthogonal to one another.

18. Define Rayleigh quotient. State Rayleigh's principle and also prove the theorem.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. (a) (i) Under what conditions is the following product nonsingular?

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ -2 & 0 & d_3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(ii) Solve the system $Ax = b$ starting with $Lc = b$:

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = b$$

(b) Find L and U for the nonsymmetric matrix

$$A = \begin{bmatrix} a & r & r & r \\ a & b & s & s \\ a & b & c & t \\ a & b & c & d \end{bmatrix}$$

Find the four conditions on a, b, c, d, r, s, t to get $A = LU$ with four pivots.

20. (i) Write the complete solution $x = x_p + x_n$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

(ii) Find the dimension and a basis for the four fundamental subspaces for the following matrix.

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$



21. (i) Find the determinant of

$$(a) A = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}$$

$$(b) A = \begin{bmatrix} 4 & 4 & 8 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$(c) B = U^T$$

$$(d) C = U^{-1}$$

(ii) (a) Evaluate the following determinant by using cofactor of row 1.

$$A = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{bmatrix}$$

(b) Subtract column 1 from the other columns and recompute.

22. (i) Find the SVD decomposition of

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$$

(ii) Decide whether the following matrices are positive definite.

$$A = \begin{bmatrix} 1 & 5 \\ 4 & 7 \end{bmatrix}, B = \begin{bmatrix} -2 & 4 \\ -1 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 \\ 5, 2 \end{bmatrix}, D = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

(2×5=10 weightage)

