



QP CODE: 25050779



Reg No : .....

Name : .....

## MSc DEGREE (CSS) EXAMINATION DECEMBER 2025

### First Semester

MSc Statistics with Data Science

### Core Courses - ST040103 - ANALYTICAL METHODS FOR STATISTICS

2020 Admission Onwards

26C57799

Time: 3 Hours

Weightage: 30

#### Part A (Short Answer Questions)

Answer any **eight** questions.

Weight **1** each.

1. Show that every bounded sequence with a unique limit point is convergent.
2. Find the Laplace transform of the real valued functions  $e^{at}$ ,  $\cos(at)$ .
3. Show that for a real sequence,  $\{a_n\}$ ,  $\inf(a_n) \leq \underline{\lim}(a_n) \leq \overline{\lim}(a_n) \leq \sup(a_n)$
4. Show that the system of non-null mutually orthogonal vectors is linearly independent.
5. What do you mean by the orthonormal basis of a vector space?
6. Distinguish between row space and column space.
7. Define characteristic vectors and characteristic sub-space of a square matrix.
8. Show that the g inverse of transpose of a matrix is the transpose of the g inverse of the matrix.
9. Define the linear transformation of quadratic forms. What do you mean by the matrix of transformation?
10. Distinguish between index and signature of quadratic forms.

(8×1=8 weightage)

#### Part B (Short Essay/Problems)

Answer any **six** questions.

Weight **2** each.

11. Consider the sequence  $\{a_n\}$  where  $a_n = \sin\left(\frac{n\pi}{2}\right) + \frac{(-1)^n}{n}$  then show that  $\underline{\lim}(a_n) = -1$  and  $\overline{\lim}(a_n) = 1$
12. State and prove Taylor's theorem.
13. Illustrate transpose of a matrix. State and prove any 3 properties of matrix transpose.
14. Define linear independence of vectors and examine the linear independence of the following vectors-
  - $(1, 0, -1), (1, 1, 1)$
  - $(1, 2, 1), (1, 1, 1)$





15. Show that the characteristic roots of a skew symmetric matrix are either 0 or purely imaginary.
16. State and prove the necessary and sufficient condition for the existence of g inverse of a matrix.
17. Deduce  $3x^2 + 5y^2 + 4z^2 + 2xy + 3xz + 4yz$  using Lagrange's reduction method
18. Describe spectral matrices. Show that the spectral matrices are symmetric and idempotent.

(6×2=12 weightage)

### Part C (Essay Type Questions)

Answer any **two** questions.

Weight **5** each.

19.
  - State the necessary and sufficient condition for the convergence of a monotonic sequence.
  - Show that the sequence  $\{a_n\}$  defined by  $a_{n+1} = \frac{1}{2} \left( a_n + \frac{9}{a_n} \right)$  and  $a_1 > 0$  is convergent and the limit of the sequence is 3.
20.
  - Define orthonormal basis of a vector space.
  - Construct an orthonormal set of vectors from the following vectors  $X = [1, 1, 0]'$ ,  $Y = [0, 1, 1]'$ ,  $Z = [1, 0, 1]'$ .
21.
  - Define algebraic multiplicity and geometric multiplicity.
  - Show that the geometric multiplicity of a characteristic root cannot exceed the algebraic multiplicity.
22.
  - Show that the following matrix,  $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$  is similar to a diagonal matrix.
  - Write down the transforming matrix and diagonal matrix.

(2×5=10 weightage)

