



QP CODE: 25050800



25050800

Reg No :

Name :

MSc DEGREE (CSS) EXAMINATION DECEMBER 2025

First Semester

MSc Statistics with Data Science

Core Courses - ST040101 - MEASURE AND PROBABILITY THEORY

2020 Admission Onwards

0D08D235

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight **1** each.

1. Find the limit, if exists, of the sequence of set $A_n = \left(1 - \frac{1}{n}, 3 + \frac{1}{n}\right), n = 1, 2, \dots$
2. Explain a sigma finite measure.
3. Prove or disprove: If f is a measurable function, then $|f|$ is also measurable.
4. Define pairwise independence and mutual independence.
5. Define a random variable
6. What do you mean by degenerate distribution? Give its probability mass function and hence obtain its distribution function.
7. Define expectation of arbitrary random variables.
8. State Markov's inequality.
9. State Borel-Cantelli Lemma.
10. Show that Gamma distribution with finite variance converges to normal distributions using central limit theorem.

.(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any **six** questions.

Weight **2** each.

11. Prove that the intersection of an arbitrary number of sigma fields is a sigma field. Comment on the union of two sigma fields.
12. Prove that a measurable function is integrable if and only if $|f|$ is integrable and $\left| \int_{\Omega} f d\mu \right| \leq \int_{\Omega} |f| d\mu$.





13. Give three definitions of probability with their advantages and disadvantages.
14. Define independence of two events. If A and B are independent events, show that A^c and B^c are also independent.
15. Prove that $X_n \xrightarrow{r} X \Rightarrow X_n \xrightarrow{P} X$ Also prove that if the X_n 's are almost surely bounded, Conversely $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{r} X$ for all r .
16. Derive the probability density function of the distribution whose characteristic function given by $\phi_X(t) = e^{it\mu - \frac{t^2\sigma^2}{2}}$
17. Show that every sequence of independent random variables with uniformly bounded variances obeys the strong law of large numbers.
18. State Liapounov's central limit theorem and Lindberg-Feller central limit theorem

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. State and prove Fatou's lemma.

20.

1. State and prove Jordan – Decomposition theorem.

2. Let X be r.v with distribution function $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2} & \text{if } x = 0 \\ \frac{1}{2} + \frac{x}{2} & \text{if } 0 < x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$. Obtain the representation of F as $F = \alpha F_d + (1 - \alpha) F_c$, where F_d is the discrete distribution function and F_c is the continuous distribution function

21.

1. State and prove Holders's Inequality
2. State and prove C_r inequality

22. State and prove Khintchine's weak law of large numbers.

(2×5=10 weightage)

