

QP CODE: 25050628



Reg No :

Name :

MSc DEGREE (CSS) EXAMINATION, DECEMBER 2025

First Semester

M.Sc STATISTICS (APPLIED)

Core Courses - ST030101 - PROBABILITY THEORY

2019 Admission Onwards

73F70543

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight **1** each.

1. Examine whether limit of the sequence $A_n = \left(1, 2 + \frac{1}{n}\right)$, $n = 1, 2, \dots$ exists. If so obtain the limit.
2. Explain Borel field and σ -field.
3. Define independence of two events. If A and B are independent events, show that A and B^c are also independent.
4. State Markov inequality and Tchebychev's inequality.
5. State the inversion formula for integer valued random variables.
6. Define convergence in rth mean.
7. Define complete convergence of distributions.
8. State Lindberg-Levy Central Limit Theorem..
9. State Khinchines Weak Law of Large Numbers.
10. State Kolmogorov strong law of large numbers for i.i.d random variables.

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any **six** questions.

Weight **2** each.

11. Find $\lim_{n \rightarrow \infty} P\left[47 - \frac{2}{n} < X < 60 + \frac{4}{n}\right]$ where X follows Normal distribution with mean 55 and S.D. 3. State the result you used.
12. Distinguish between conditional probability and independence of two events. In the usual notation, prove that $P(A) = \sum_i P(B_i)P(A/B_i)$, stating the assumptions.





13. How would you define the expected value of a random variable X ? Let X be a random variable with p.d.f. $f(x) = \frac{2}{x^3}$, $1 < x < \infty$ and 0 elsewhere. Examine whether $E(X)$ and $E(X^2)$ exist.
14. Explain the concept of characteristic function for a vector random variable. Give an example. Mention its important properties.
15. If ϕ_i is a characteristic function for $i=1,2$ then show that $\frac{[\phi_1 + \phi_2]}{2}$ is also a characteristic function.
16. Explain in detail about the convergence in distribution of random variables.
17. For what values of α does strong law of large numbers hold for the sequence $\{X_n\}$ of independent r.v's $P[X_n = \pm n^\alpha] = \frac{1}{2}$.
18. Let $X_1, X_2, \dots$ be i.i.d. random variables with common p.d.f $f(x) = \frac{1}{2\sigma} e^{\frac{-|x-\mu|}{\sigma}}$, $-\infty < x < \infty$. Examine whether the sequence of random variables $\{X_k\}$ obeys CLT.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. State and prove Borel 0-1 law.
20. (a) Define distribution function. Establish a set of necessary and sufficient condition for a function to be the distribution function of a univariate random variable.
- (b) Let $F(x,y) = 0$ if $x < 0$, $y < 0$ or $x+y < 1$
 $= 1$, elsewhere
 Verify whether $F(x,y)$ is a distribution function or not.
21. (i) If $X_n \xrightarrow{p} X$ and $Y_n \xrightarrow{p} Y$, then show that $X_n + Y_n \xrightarrow{p} X + Y$.
- (ii) If $X_n \xrightarrow{p} X$ and $Y_n \xrightarrow{p} Y$, then show that $X_n Y_n \xrightarrow{p} XY$.
- (iii) Show that $X_n \xrightarrow{p} X \Rightarrow X_n - X_m \xrightarrow{p} 0$, as $n, m \rightarrow \infty$.
22. Examine whether Central Limit Theorem holds for the sequence of independent random variables $\{X_k\}$ such that:
- (i) $P[X_k = k] = P[X_k = -k] = k^{-\lambda}/2$
 $, P[X_k = 0] = 1 - k^{-\lambda}, \lambda > 0$.
- (ii) $P[X_k = k] = P[X_k = -k] = \frac{1}{2\sqrt{k}}, P[X_k = 0] = 1 - \frac{1}{\sqrt{k}}, n = 1, 2, \dots$

(2×5=10 weightage)

