

QP CODE: 25050659



Reg No :

Name :

MSc DEGREE (CSS) EXAMINATION, DECEMBER 2025

First Semester

M Sc STATISTICS

Core Courses - ST010101 - PROBABILITY AND MEASURE THEORY

2019 Admission Onwards

EC5840A7

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

Weight 1 each.

1. Define limit inferior and limit superior of a sequence of events. By an example show that they can be different.
2. Define field and sigma field.
3. Explain integral of a non-negative measurable function over a measurable set.
4. Define discrete probability space .Give an example.
5. Define distribution function of a two dimensional random variable and mention its important properties.
6. Define Expectation of a random variable. Show that $E(X+Y) = E(X) + E(Y)$.
7. State and prove Jensen inequality.
8. Define almost sure convergence
9. Define convergence in rth mean.
10. Define complete convergence of distributions.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

Weight 2 each.

11. For a finite measure μ , show that $\mu(\lim A_n) = \lim(\mu(A_n))$.
12. Define a Lebesgue measure on R. Show that the Lebesgue measure of the set of rationals in R is zero.
13. Explain the basic properties of Lebesgue integral.





14. Distinguish between conditional probability and independence of two events. In the usual notation, prove that $P(A) = \sum_i P(B_i) P(A/B_i)$, stating the assumptions.
15. If X and Y are independent random variables, show that $E(XY) = E(X)E(Y)$, whenever both sides exists.
16. If $\{A_n\}$ be a sequence of independent events of a Borel field and $\sum_1^\infty P(A_n) = \infty$, show that $P(\limsup A_n) = 1$
17. Show that $X_n \xrightarrow{L} X, Y_n \xrightarrow{L} C \Rightarrow \frac{X_n}{Y_n} \xrightarrow{L} \frac{X}{C}, (C \neq 0), [Y_n = 0] = \emptyset$.
18. Show that $X_n \xrightarrow{p} X \Rightarrow X_n \xrightarrow{d} X$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. (a) Define Lebesgue-Stieltje's measure.
- (b) Let F be the distribution function on $\mathbb{R}$ given by

$$\begin{aligned} F(x) &= 0, & \text{if } x < -1 \\ &= 1+x, & \text{if } -1 \leq x < 0 \\ &= 2+x^2, & \text{if } 0 \leq x < 2 \\ &= 10, & \text{if } x \geq 2 \end{aligned}$$

If μ is the Lebesgue-Stieltje's measure induced by F , compute the measure of the sets:

- (i) $\{2\}$, (ii) $[-1/2, 3]$, (iii) $\{x: |x| + 2x^2 > 1\}$, (iv) $\left[0, \frac{1}{2}\right] \cup (1, 2]$
20. (a) State and prove Lebesgue dominated convergence theorem.
- (b) State and prove Fatou's lemma. Deduce monotone convergence theorem.
21. Let $\Omega = [0, 2]$. The

σ -field of events be the class generated by $\left\{\left[0, \frac{2}{3}\right], \left[0, \frac{3}{2}\right]\right\}$. Examine whether $X(\cdot)$, $Y(\cdot)$, $Z(\cdot)$ defined below is a random variable

$$\begin{aligned} (i) X(\omega) &= 1, \text{ if } 0 \leq \omega \leq \frac{2}{3} \\ &= -3, \text{ if } \frac{2}{3} < \omega \leq \frac{3}{2} \\ &= 0, \text{ if } \frac{3}{2} < \omega \leq 2 \\ (ii) Y(\omega) &= 2, \text{ if } 0 \leq \omega < \frac{3}{2} \\ &= 4, \text{ if } \frac{3}{2} \leq \omega \leq 2 \end{aligned}$$





$$(iii) Z(\omega) = -8, \text{ if } 0 \leq \omega < \frac{2}{3}$$

$$= 0, \text{ if } \frac{3}{2} < \omega \leq 2$$

22. (i) State and prove Cr- inequality . (ii) State and prove Markov inequality and hence deduce Tchebychev's inequality.

(2×5=10 weightage)

