QP CODE: 25047273

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Reg No : .....

Name : .....

M.Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2025

Third Semester

M Sc MATHEMATICS

Core Course - ME010302 - PARTIAL DIFFERENTIAL EQUATIONS

2019 ADMISSION ONWARDS

9E5E0B84

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the integral curves of $\frac{x\,dx}{y^2z} = \frac{dy}{xz} = \frac{dz}{y^2}$
2. Eliminate the arbitrary function f from the equation $f(x^2+y^2+z^2,\ z^2-2xy)=0$
3. Explain the different types of integrals of a nonlinear partial differential equations of first order.
4. Show that the equations $f(x,y,z,p,q)=0,\ g(x,y,z,p,q)=0$ are compatible if $\frac{\partial(f,g)}{\partial(x,p)}+\frac{\partial(f,g)}{\partial(y,q)}=0.$
5. Fina a complete integral of the equation $p+q=pq.$
6. If $z=f(x+ay)+g(x-ay)$, show that $t=a^2r.$
7. Show that $F(D,D')e^{ax+by}\phi(x,y)=e^{ax+by}F(D+a,D'+b)\phi(x,y).$
8. Find the particular integral of $[D^2-D']z=e^{x+y}.$
9. Establish a formula for finding the potential function of a family of equipotential surfaces.
10. Show that that $\psi=\frac{q}{|r-r'|}$ where q is a constant and r and r' are the position vectors of the points (x,y,z) and (x',y',z'), is a solution of the Laplace's equation $\nabla^2\psi=0$

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Find the orthogonal trajectories on the sphere $x^2+y^2+z^2=a^2$ of its intersections with the paraboloids $xy=cz,\ c$ being a parameter.
12. Verify that the equation $yz(y+z)\,dx+xz(x+z)\,dy+xy(x+y)\,dz=0$ is integrable and solve it.
13. Solve the linear partial differential equation $(y+zx)p-(x+yz)q=x^2-y^2.$
14. Find the complete integral of the equation $px^5-4q^3x^2+6x^2z-2=0.$

15. Solve $[D^4 + D'^4]z = 2D^2D'^2z$.

16. Reduce the equation to canonical form $u_{xx} + x^2u_{yy} = 0$.

17. Prove that if $f(x, y, z) = c$ is a family of equipotential surfaces, then $\frac{\nabla^2 f}{|\nabla f|^2}$ is a function of f alone.

18. Prove that the function $\phi = sinxcoshy + 2cosxsinhy + x^2 - y^2 + 4xy$ is Harmonic and find the corresponding analytic function $\phi + i\psi$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Prove that a Pfaffian differential equation in two variables always possesses an integrating factor.
b) Prove that necessary and sufficient condition that there exists between two functions $u(x, y)$ and $v(x, y)$ a relation $F(u, v) = 0$ not involving x or y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$.

20. Find the general equation of the surfaces orthogonal to the family given by $x(x^2 + y^2 + z^2) = c_1y^2$ showing that one such orthogonal set consists of the the family of spheres given by $x^2 + y^2 + z^2 = c_2z$. If a family exists, orthogonal to both the above equations, show that it must satisfy $2x(x^2 - z^2)dx + y(3x^2 + y^2 - z^2)dy + 2z(2x^2 + y^2)dz = 0$.

21. By Jacobi's method, solve $z^2 + zu_z - u_x^2 - u_y^2 = 0$.

22. By separating the variables solve the one- dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$

(2×5=10 weightage)