



QP CODE: 25050458



25050458

Reg No :

Name :

MSc DEGREE (CSS) EXAMINATION DECEMBER 2025

First Semester

M Sc MATHEMATICS

Core Courses - ME010105 - GRAPH THEORY

2019 Admission Onwards

DA350B30

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

Weight 1 each.

1. Define (a) graph (b) isomorphism of graphs (c) simple graph (d) induced subgraph of a graph
2. Show that the edges incident at a vertex of a graph G give rise to a complete subgraph of the line graph $L(G)$.
3. State Whitney's characterization theorem of 2-connected graph and using this prove that a graph G with atleast three vertices is 2-connected if and only if any two vertices of G lie on a common cycle.
4. Define (a) cyclical edge connectivity of a graph
(b) nonseparable graph
(c) block of a graph
(d) End block of a graph
5. Write Kruskal's algorithm for determining a minimum weight spanning tree in a connected weighted graph.
6. Explain Knight's Tour in a Chessboard.
7. Define chromatic number and chromatic partition of vertices of a graph G .
8. State Brook's theorem.
9. Define a planar graph.
10. What is the spectrum of K_n

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

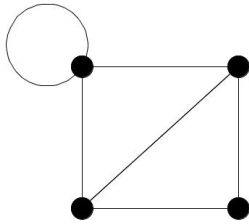
Weight 2 each.

11. Define graphical sequence and write the necessary condition for $d = \{d_1, d_2, \dots, d_n\}$ to be graphical. Show that $d = \{7, 6, 3, 3, 2, 1, 1, 1\}$ is not graphical.
12. Find the order and size of $G_1 \times G_2$





13. Show that if a simple cubic connected graph has a cut vertex then it has a cut edge.
14. Find the number of spanning trees of the following graph



15. Explain Konigsberg Bridge Problem.
16. For any simple graph G , prove that $2\sqrt{n} \leq \chi + \chi^c \leq n + 1$ and $n \leq \chi\chi^c \leq ((n + 1)/2)^2$
17. If G is simple connected planar bipartite graph with $n \geq 3$, then show that $m \leq 2n - 4$
18. Show that $K_{3,3}$ is a nonplanar graph using the Jordan Curve Theorem.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. 1. Show that every vertex of a disconnected tournament T on n vertices with $n \geq 3$ is contained in a directed k -cycle, $3 \leq k \leq n$.
 2. Show that for every positive integer $n \geq 3$ there exist a tournament not containing a spanning directed cycle.
 3. Define (i) Directed graph
 (ii) outneighbour of a vertex and inneighbour of a vertex
 (iii) dicomponent
 (iv) tournament
20. 1. Show that every tree has a centre consisting of either a single vertex or two adjacent vertices.
 2. Prove that a simple graph G is a tree if and only if any two distinct vertices are connected by a unique path.
 3. Show that a connected graph G is a tree if and only if every edge of G is a cut edge.
21. (a) Define closure of a graph. Prove that a graph has one and only one closure.
 (b) Prove that if $cl(G)$ is hamiltonian then G is hamiltonian.
 (c) Give an example of a graph G such that $cl(G) = G$
22. State and prove Heawood's five color theorem.

(2×5=10 weightage)

