

QP CODE: 25050452



Reg No :

Name :

MSc DEGREE (CSS) EXAMINATION, DECEMBER 2025

First Semester

M Sc MATHEMATICS

Core Courses - ME010102 - LINEAR ALGEBRA

2019 Admission Onwards

ECE56C7C

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight **1** each.

1. If C is the field of complex numbers, which vectors in C^3 are linear combinations of $(1,0,-1)$, $(0,1,1)$ and $(1,1,1)$?
2. Let S be a linearly independent subset of a vector space V . Suppose β is a vector in V which is not in the subspace spanned by S . Then show that the set $S \cup \{\beta\}$ is linearly independent.
3. Check whether the operator T on $F^{m \times n}$ defined by $T(A) = PAQ$ is linear, where $P \in F^{m \times n}$ and $Q \in F^{n \times n}$ are fixed matrices over the field F .
4. Prove that the null space of any nonzero linear functional on a finite dimensional vector space V is a hyper space of V .
5. Let A be an $n \times n$ matrix with real entries. If $A = 0$ prove that $\text{trace}(A^T A) = 0$.
6. Show that a linear combination of n -linear functions is n -linear.
7. If A is an invertible $n \times n$ matrix over a field, show that $\det A \neq 0$.
8. Let K be the ring of rational numbers and let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Is A invertible as a matrix over K ? If so, find the inverse.
9. Find the characteristic values, if any, of the linear operator T on R^2 which is represented in the standard ordered basis by the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.
10. Define minimal polynomial for a linear operator on a finite dimensional vector space V . What is the minimal polynomial for the zero operator?

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any **six** questions.





11. Define subspace spanned by a set of vectors in a vector space V . Show that the subspace spanned by a non-empty subset S of a vector space V is the set of all linear combinations of vectors in S .
12. Find the coordinate matrix of the vector $(1, 0, 1)$ in the basis of $\mathbb{C}^3$ consisting of the vectors $(2i, 1, 0)$, $(2, -1, 1)$, $(0, 1 + i, 1 - i)$, in that order.
13. Prove that every n -dimensional vector space V over the field F is isomorphic to the space F^n .
14. If V is a finite dimensional vector space and T is a linear operator on V prove that T is invertible if and only if $[T]_B$ is invertible, where B is any ordered basis for V .
15. If S is any subset of a finite dimensional vector space V and W is the subspace of V spanned by S then prove that $S^{00} = W$.
16. If A and B are two $n \times n$ matrices over K , then show that $\det(AB) = (\det A)(\det B)$.
17. Let T be a linear operator on $\mathbb{R}^3$ which is represented in the standard ordered basis by the matrix

$$A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}. \text{ Prove that } T \text{ is diagonalizable by exhibiting a basis for } \mathbb{R}^3, \text{ each vector of which is a characteristic vector of } T$$

18. If $E_1, E_2, \dots, E_k$ are k linear operators on V which satisfy the conditions

- i. Each E_i is a projection
- ii. $E_i E_j = 0$ if $i \neq j$
- iii. $I = E_1 + \dots + E_k$

and let W_i be the range of E_i then prove that $V = W_1 \oplus \dots \oplus W_k$

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19.

Consider the 5×5 matrix $A = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 1 & 2 & -1 & -1 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 2 & 4 & 1 & 10 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

- (a) Find an invertible matrix P such that PA is a row-reduced echelon matrix R .
- (b) Find a basis for the row space W of A .
- (c) Which vectors $(b_1, b_2, b_3, b_4, b_5)$ are in W ?
- (d) Find the coordinate matrix of each vector $(b_1, b_2, b_3, b_4, b_5)$ in W in the ordered basis chosen in (b).



- (e) Write each vector $(b_1, b_2, b_3, b_4, b_5)$ in W as a linear combination of the rows of A .
- (f) Give an explicit description of the vector space V of all 5×1 column matrices X such that $AX = 0$.
- (g) Find a basis for V .

20. Let V and W be finite dimensional vector spaces over the field F such that $\dim V = \dim W$. If $T: V \rightarrow W$ be a linear transformation, prove that the following statements are equivalent:

- (i) T is invertible.
- (ii) T is non-singular.
- (iii) T is onto.
- (iv) If $\{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$ is a basis for V then $\{T\alpha_1, T\alpha_2, T\alpha_3, \dots, T\alpha_n\}$ is a basis for W .
- (v) There is some basis $\{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$ for V such that $\{T\alpha_1, T\alpha_2, T\alpha_3, \dots, T\alpha_n\}$ is a basis for W .

21. (a) Let $n > 1$ and let D be an alternating $(n-1)$ -linear function on $(n-1) \times (n-1)$ matrices over K . For each

$j, 1 \leq j \leq n$, show that the function E_j defined by
$$E_j(A) = \sum_{i=1}^n (-1)^{i+j} A_{ij} D_{ij}(A)$$
 is an alternating n -linear function on $n \times n$ matrices A . If D is a determinant function, then show that so is each E_j .

(b) If $K = \mathbb{R}[x]$ and $A = \begin{bmatrix} x-1 & x^2 & x^3 \\ 0 & x-2 & 1 \\ 0 & 0 & x-3 \end{bmatrix}$, find $E_j(A)$ for each j .

22. 1. Let V be a finite dimensional vector space over the field F and let T be a linear operator on V . Prove that T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1)(x - c_2) \cdots (x - c_k)$ where $c_1, c_2, \dots, c_k$ are distinct elements of F .
2. Every matrix A such that $A^2 = A$ is similar to a diagonal matrix

(2×5=10 weightage)