



MSc DEGREE (CSS) EXAMINATION, DECEMBER 2025

First Semester

M Sc MATHEMATICS

Core Courses - ME010101 - ABSTRACT ALGEBRA

2019 Admission Onwards

5C247BAF

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight 1 each.

- List all the elements of finite order of the group $\mathbb{Z}_2 \times \mathbb{Z} \times \mathbb{Z}_4$. Justify your answer.
- Prove that if m is a square free integer, then every abelian group of order m is cyclic.
- Let H be a subgroup of a group G . Prove that G is an H -set under the action $*(h, g) = hg$ for $h \in H$ and $g \in G$.
- Find the kernel of the homomorphism $\phi: \mathbb{Z}_{12} \rightarrow \mathbb{Z}_3$, where $\phi(1) = 2$.
- Describe class equation of a finite group G .
- Show that every group of order 45 has a normal subgroup of order 9.
- Find $f(x)g(x)$ in $\mathbb{Z}_7$, where $f(x) = x^3 + 2x^2 + 5$ and $g(x) = 3x^2 + 2x$.
- Check $8x^3 + 6x^2 - 9x + 24$ is irreducible over $\mathbb{Q}$.
- Define Quaternion ring.
- Let F be the ring of all functions mapping $\mathbb{R}$ into $\mathbb{R}$ and let N be the subring of all functions f such that $f(2) = 0$. Is N an ideal in F ? Why or Why not?

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any **six** questions.

Weight 2 each.

- Let H be a normal subgroup of a group G . Prove that the cosets of H form a group G/H under the binary operation $(aH)(bH) = (ab)H$.





12. Let G be a finite group and X a finite G -set. If r is the number of orbits in X under G , prove that $r \cdot |G| = \sum_{g \in G} |X_g|$
13. Let G be a finite group and let $|G| = p^n m$, where $n \geq 1$ and p does not divide m . Prove that G contains a subgroup of order p^i , for each i , where $1 \leq i \leq n$.
14. Prove that no group of order 36 is simple.
15. Show that for every integer n , the number $n^{33} - n$ is divisible by 15
16. Show that for $[(a,b)]$ and $[(c,d)]$ in F the equations $[(a,b)] + [(c,d)] = [(ad+bc, bd)]$ and $[(a,b)] [(c,d)] = [(ac, bd)]$ give well defined addition and multiplication on F where F be the desired field of quotients of an integral domain D .
17. Show that if R, R' and R'' are rings and if $\phi : R \rightarrow R'$ and $\psi : R' \rightarrow R''$ are homomorphisms, then the composite function $\psi \circ \phi : R \rightarrow R''$ is a homomorphism.
18. If R is a ring with unity 1, then show that the map $\phi : \mathbb{Z} \rightarrow R$ given by $\phi(n) = n \cdot 1$ for $n \in \mathbb{Z}$ is a homomorphism of $\mathbb{Z}$ into R .

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. (a) Let X be a G -set. Prove that $\{g \in G / gx = x\}$ is a subgroup of G for each $x \in X$.
(b) Let X be a G -set and let $x \in X$. Prove that $|Gx| = (G : G_x)$.
20. (a) Let p be a prime. Let G be a finite group and let p divide $|G|$. Prove that G has a subgroup of order p .
(b) Let G be a finite group. Prove that G is a p -group if and only if $|G|$ is a power of p .
21. (a) Explain the division algorithm in $F[x]$.
(b) If G is a finite subgroup of the multiplicative group of a field F , then prove that G is cyclic.
22. Define
 - (a) Endomorphism ring
 - (b) Group ring
 - (c) Ring homomorphism
 - (d) Ideal of a ring
 - (e) Factor ring
 - (f) Prime ideal
 - (g) Maximal ideal
 - (h) Principal ideal

(2×5=10 weightage)

