



25802571

QP CODE: 25802571

Reg No :

Name :

INTEGRATED MSC DEGREE EXAMINATION, MAY 2025**Fourth Semester**

INTEGRATED MSC BASIC SCIENCE-PHYSICS

**COMPLEMENTARY - IPH4CM04 - SPECIAL FUNCTIONS, LA PLACE TRANSFORMS
AND COMPLEX ANALYSIS**

2021 Admission Onwards

9631B182

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)*Answer any **eight** questions.**Weight 1 each.*

1. Give Legendre's equation.
2. What is Legendre polynomial of order n ?
3. Define the Laplace Transform of $f(t)$. Also find $L(1)$.
4. Find the Laplace Transform of the function $f(t) = 2t + 6$.
5. Find the inverse Laplace Transform of $\frac{2}{s} + \frac{1}{s+2}$
6. Define limit and continuity of a complex function $f(z)$ at $z = z_0$.
7. Find the real and imaginary parts of $\sinh z$ and $\cosh z$.
8. Define the natural logarithm $\ln(z)$ of a complex number z .
9. (a) State the Cauchy's integral theorem.
(b) If $f(z)$ is an entire function, what is the value of its integral over any simple closed curve? Hence what is $\oint_c e^z dz$.
10. Integrate $g(z) = \frac{z^2+1}{z^2-1}$ in the counter clockwise direction around a circle of radius 1 with center $z = \frac{1}{2}$.
(8×1=8 weightage)

Part B (Short Essay/Problems)*Answer any **six** questions.**Weight 2 each.*

11. Give the power series expansion of $\cos x$





12. Give the general solution of Bessel's equation for all $x \neq 0$
 13. Prove that $L[t^a] = \frac{\Gamma(a+1)}{s^{a+1}}$.
 14. Find the Laplace Transform of $t^2 e^{-3t} \sin 2t$.
 15. Find and plot all the cube roots of $8i$.
 16. Prove that $|\sin z|^2 = \sin^2 x + \sinh^2 y$.
 17. Using the ML Inequality, prove that $|\int_C \frac{1}{z^4} dz| \leq 4\sqrt{2}$ where C is the line segment from $z = i$ to $z = 1$ and observing that of all the points on this line segment the mid point is the closest.
 18. Integrate the function $f(z) = \frac{e^{3z}}{(4z - \pi i)^3}$ in the counter clockwise direction around the circle $|z| = 1$.
- (6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. Apply the binomial theorem to $(x^2 - 1)^n$ and differentiating prove that $P_n(x) = \frac{1}{n!2^n} \frac{d^n}{dx^n} (x^2 - 1)^n$
20. Evaluate (i) $L^{-1}\left[\frac{s-a}{s(s+a)}\right]$.
(ii) $L^{-1}\left[\frac{s-2}{s^2(s^2+4)}\right]$
21. Check whether the function $u = x^2 + y^2$ is harmonic or not. If yes, find a corresponding analytic function $f(z)$.
22. (a) When do we say that integral of a function $f(z)$ is independent of path in a domain D ?
(b) Explain the principle of deformation.
(c) Show that $\int_C \frac{1}{z} dz = 2\pi i$ where C is any closed contour oriented in the counter clockwise direction surrounding the origin.

(2×5=10 weightage)

