

QP CODE: 24026871



Reg No : .....

Name : .....

**B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE  
EXAMINATIONS, OCTOBER 2024**

**Third Semester**

B.Sc Statistics Model I

**Core Course - ST3CRT03 - FUNDAMENTALS OF RANDOM VARIABLES-COURSE III**

2017 Admission Onwards

A7E79F45

Time: 3 Hours

Max. Marks : 80

**Part A**

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Distinguish between probability density function and distribution function of a random variable.
2. If  $f(x, y) = k$  ;  $0 < x < 1$  and  $0 < y < 1$  is a joint pdf then determine k.
3. What are the properties of joint p.d.f.?
4. Define mathematical expectation of a continuous random variable.
5. Write the expressions for third and fourth central moments using raw moments.
6. Show that ,  
$$Cov(X, Y) = E(XY) - E(X)E(Y).$$
7. Mention any two limitations of moment generating function.
8. Mention any two properties of moment generating function.
9. State the principle of least squares.
10. State the normal equations for fitting a parabola.
11. Distinguish between positive and negative correlation.
12. If the two regression equations are  $3x+12y-10=0$  and  $3y+9x-46=0$ . Find the average values of x and y.

(10×2=20)

**Part B**





Answer any **six** questions.

Each question carries **5** marks.

13. There are seven keys in a bunch of which only one fits a lock. Keys are tried one by one at random without replacement until the key fitting the lock is selected. If  $X$  denotes the number of trials write down the p.d.f of  $X$ .
14. Two fair dice are thrown. If  $X$ = sum of the numbers thrown and  $Y$ =maximum of the numbers thrown. Write the joint pdf of  $(X, Y)$ .
15. Define stochastic independence between two variables in terms of joint distribution and marginal distribution.
16. If  $X$  and  $Y$  are random variables with joint p.d.f

$$f(x, y) = \frac{1}{3} \text{ if } (x, y) = (0, 0), (0, 1) \text{ or } (1, 1) \text{ and zero elsewhere.}$$

$$\text{Find } E\left[X - \frac{1}{3}\right] \left[Y - \frac{2}{3}\right]$$

17. Two random variables  $X$  and  $Y$  have the joint pdf  
 $f(x, y) = 6 - x - y$  ;  $0 < x < 2$  ,  $2 < y < 4$  and 0 elsewhere. Find  $E[X]$  and  $E[Y]$
18. Find the m.g.f of  
 (i)  $Y = aX + b$   
 (ii)  $Y = \frac{(X-a)}{b}$   
 (iii) the sum of two independent random variables.
19. Fit a curve of the form  $y = aebx$  to the following data  $X$ : 1 2 3 4 5 6  $Y$ : 1.7 4.5 13.5 40.2 124 280
20. Derive Spearman's rank correlation coefficient.
21. Show that regression coefficient is independent of change in origin but not in the scale.

(6×5=30)

### Part C

Answer any **two** questions.

Each question carries **15** marks.

22. Given that

$$\begin{aligned} f(x) &= k && \text{; if } x = 0, \\ &= 2k && \text{; if } x = 1 \\ &= 3k && \text{; if } x = 2 \\ &= 0 && \text{; elsewhere} \end{aligned}$$

is a pmf .

1. Find the value of  $k$ ,





2. Determine the distribution function,
3. Find the probabilities  $P[X \geq 2]$ ,  $P[X \leq 1]$  and  $P[0 < X < 2]$ ,
4. What is the smallest value of  $m$  such that  $P[X \leq m] > 0.5$ .

23. The joint p.d.f of  $X$  and  $Y$  is

$$f(x, y) = 2, \quad 0 \leq x \leq y \leq 1$$
$$= 0 \quad \text{elsewhere.}$$

Show that  $V(X|y) = \frac{y^2}{12}$

24. a) Define m.g.f of a variable  $X$ . Hence find the m.g.f of  $Y = Ax + b$   
b) If  $X$  and  $Y$  are independent, prove that the m.g.f of  $X + Y$  is equal to the product of the m.g.f.s of  $X$  and  $Y$ .
25. The ranks of same 16 students in Mathematics and Statistics are as follows. Two numbers within brackets denote the ranks of the students in Mathematics and Statistics:
- (1,1) (2,10) (3,3) (4,4) (5,5) (6,7) (7,2) (8,6) (9,8) (10,11)  
(11,15) (12,9) (13,14) (14,12) (15,16) (16,13)
- Calculate the rank correlation coefficient for proficiencies of this group in Mathematics and Statistics.

(2×15=30)

