QP CODE: 24027568

24027568

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, OCTOBER 2024

Third Semester

B.Sc Statistics Model I

COMPLEMENTARY COURSE - MM3CMT05 - MATHEMATICS - VECTOR CALCULUS,DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORM

2017 Admission Onwards

8C37EECF

Time: 3 Hours — Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Are the vectors [4, 2, 9],[3, 2, 1], [-4, 6, 9] linearly independent ?
2. Define the derivative of a vector function.
3. Calculate the Laplacian of $f(x, y, z) = x^2y^2 \sin z$.
4. Find the curl of $\vec{v} = (x^2 - y)\hat{i} + 4z\hat{j} + x^2\hat{k}$
5. Show that $\dfrac{dy}{dx} = \dfrac{y}{x} + \sin\dfrac{y}{x}$ is a homogeneous differential equation.
6. What transformation reduces the differential equation $\dfrac{dy}{dx} = \dfrac{y+x-2}{y-x-4}$ into a homogeneous equation?
7. What transformation reduces the Bernoulli's equation $x\dfrac{dy}{dx} + y = x^3y^6$ into a linear equation?
8. Solve the differential equation $y\,dx - x\,dy = x^2y^2\,dx$.
9. Write the form of Lagrange's Linear equation. Give an example.
10. Write any one method to solve the simultaeous differential equation $\dfrac{dx}{P} = \dfrac{dy}{Q} = \dfrac{dz}{R}$ where $P,\ Q$ and R are functions of $x,\ y$ and z.

11. Find $L\{e^{3t}\sin 2t\}$.

12. Find $L^{-1}\{\frac{1}{s^2-6s+13}\}$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Find the angles of the triangle ABC if the vertices are A(0, 0, 0), B(4, 2,1) and C(1, 2, 4).

14. Find the tangent to the ellipse $\frac{1}{4}x^2 + y^2 = 1$ at $P(\sqrt{2}, \frac{1}{\sqrt{2}})$.

15. Find the divergence of $\vec{v} = (x^2 + y^2 + z^2)^{-3/2}(\sin x\hat{i} + \cos y\hat{j} + \sin z\hat{k})$.

16. Find the differential equation of the family of curves given by $y = ax^3 + bx^2$ where a and b are arbitrary constants.

17. Solve the differential equation $\left(1 + y^2\right)\ dx = \left(\tan^{-1}y - x\right)\ dy$.

18. Solve the differential equation $\left(1 + e^{\frac{x}{y}}\right)\ dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)\ dy = 0$.

19. Find the partial differential equation of all spheres having their centres on the z-axis.

20. Form a partial differential equation by eliminating the arbitrary functions f and g from the relation $z = f\left(x + ay\right) + g\left(x - ay\right)$.

21. Find $L\{\sin^2 3t\}$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a) Are the vectors [4,2,9], [3,2,1], [-4,6,9] linearly independent.
    (b) For any twice continuously differentiable scalar function f , prove that $curl(gradf) = 0$ and prove it for $f = 4x^2 + 9y^2 + z^2$.
    (c) Find the directional derivative of $f = xyz$ at P(-1,1,3) in the direction of $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$.

23. (a) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{y - x + 1}{x + y - 5}$.
    (b) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{x + y + 1}{2x + 2y + 3}$.

24. (a) Solve the partial differential equation $z\,(xp - yq) = y^2 - x^2$.
(b) Solve the partial differential equation $x^2\,p + y^2\,q = (x+y)\,z$.
(c) Solve the partial differential equation $(y-z)\;p + (z-x)\;q = (x-y)$.

25. (a) State and prove first shifting theorem.
(b) Find $L^{-1}\{\frac{21s-33}{(s+1)(s-2)^3}\}$.

(2×15=30)