QP CODE: 24026882

24026882

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, OCTOBER 2024

Third Semester

COMPLEMENTARY COURSE - MM3CMT01 - MATHEMATICS - VECTOR CALCULUS, ANALYTIC GEOMETRY AND ABSTRACT ALGEBRA

Common to B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III, B.Sc Geology and Water Management Model III, B.Sc Geology Model I, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance

2017 Admission Onwards

5C62E8D6

Time: 3 Hours Max. Marks : 80

core

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the velocity and acceleration of the particle whose position is given by $r(t) = (t+1)i + (t^2-1)j$ at $t = 1$.
2. Find the arc length parametrisation of the curve $r(t) = cost\ i + sint\ j + tk$ with base point $(1, 0, 0)$.
3. Find the gradient of $f(x, y, z) = x^2 + y^2 + z^2$at $(1, 1, 1)$.
4. Define the flux of a vector field $\mathbf{F}$ across a closed curve C in space.
5. Define a conservative vector field.
6. Find the curl of the vector field $\mathbf{F} = x^2\, i + 4xy^3 j + y^2 xk$
7. Find the equation of the parabola with axis $y = 0$, vertex $(0, 0)$ and passes through the point $(3, 1)$.
8. Write the standard form of the equation of an ellipse. Also write the co-ordinates of the vertices.
9. Find the eccentricity of the ellipse $2x^2 + y^2 = 4$.

10. Find the identity element in $< Q, * >$ where $a * b = \frac{ab}{2}$ for $a, b \in Q$.

11. How many group you can define on the set G if $G = \{a, b\}$.

12. Find the remainder when -38 is divided by 7 according to division algorithm.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Find the derivative of $f(x, y) = 2x^2 + y^2$ at $(-1, 1)$ in the direction of the vector $u = 3i - 4j$.

14. Find the equation of tangent to curve $x^2 - y = 1$ at the point $(\sqrt{2}, 1)$.

15. Apply Green's Theorem to evaluate $\oint_C x \cos y \, dx - y \sin x \, dy$ where C is the square with vertices $(0, 0)$, $(\pi, 0)$, (π, π), $(0, \pi)$ oriented counterclockwise.

16. Find the area of the ellipse cut from the plane $z = cx$ where c is a constant by the cylinder $x^2 + y^2 = 1$.

17. Find the surface area of the cone $z = \sqrt{x^2 + y^2}$ for $0 \leq z \leq 1$.

18. Describe the graph of $r^2 = 4r \sin \theta$.

19. Find the vertices, focii, length of the semimajor axis and the length of the semiminor axis of the hyperbola $9y^2 - x^2 = 9$.

20. Show that if $a \in G$, where G is a finite group with identity e, then there is an $n \in Z^+$ such that $a^n = e$.

21. Show that every cyclic group with more than two elements has atleast two generators.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a) Find the unit tangent, principal normal and curvature of the curve $r(t) = (2t + 3)i + (5 - t^2)j$.
(b) Find the directions in which $f(x, y, z) = x^3 - xy^2 - z$ increases most rapidly and decreases most rapidly at the point $(1, 1, 0)$.

23. Verify Stoke's Theorem for $\mathbf{F} = (z - y)i + (z + x)j - (x + y)k$ where S is the portion of the paraboloid $z = 4 - x^2 - y^2$ above the xy-plane with upward orientation.

24. (a) Find the equation of the hyperbola when $16x^2 - 8y^2 = 16$ is shifted 1 units to the left and 3 units up. Also find the center,vertices, foci and directrix of the new hyperbola.

Sketch the new hyperbola with all these details.

(b) Find the polar equation of the circle $(x-4)^2+(y-3)^2=49$.

25. Show that the set of all permutations on a three element set forms a non abelian group under permutation multiplication and also draw its subgroup diagram.

(2×15=30)