QP CODE: 24900175

Reg No:.....

Name:.....

MAHATMA GANDHI UNIVERSITY, KOTTAYAM

FIRST SEMESTER MGU-UGP (HONOURS) REGULAR EXAMINATION NOVEMBER 2024

First Semester

Discipline Specific Core Course - MG1DSCMAT100 - GROUND ROOTS OF MATHEMATICS

(2024 ADMISSION ONWARDS)

Duration: 2 Hours Maximum Marks: 70

Remember (K), Understand (U), Apply (A), Analyse (An), Evaluate (E), Create (C), Interest (I), Appreciation (Ap), and Skill (S)

Students should attempt atleast one question from each course outcome to enhance their overall outcome attainability.

[Learning Domain][CO No(s)]

Part A
Short Answer Type Questions
Answer any five questions. Each question carries 2 marks

1	State De Morgan's laws for quantifiers.	[K]	[1]
2	State and prove identity laws of logical equivalence.	[U]	[1]
3	Define composite of two functions and explain how to find its domain.	[U]	[2]
4	Define odd function. Give an example.	[K]	[2]
5	Find $\frac{dy}{dx} if\ y = \sin^2 x$	[A]	[4]
6	Find $f'(x) if\ f(x) = \sin(2x)$.	[U]	[4]
7	Check whether $\lim_{x \to 0^+} x \ln x$ is an indeterminate form or not.	[A]	[6]
8	Illustrate decreasing functions with an example.	[U]	[5]

(5 × 2 = 10)

Part B
Short Essay Type Questions
Answer any five questions. Each question carries 6 marks

9 Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals. [U] [1]

a) $\forall x(R(x) \rightarrow H(x))$

b) $\forall x(R(x) \bigwedge H(x))$

10 Explain universal quantifier, existential quantifier and the uniqueness quantifier with examples. [K] [1]

11 Find $\lim_{x \to +\infty} \frac{x^2}{e^{3x}}$ [A] [6]

12 (i) Define the greatest integer function and the the least integer function. [U] [2]

(ii) What real numbers x satisfy the equation $\lfloor x \rfloor = \lceil x \rceil$?

13 Find $\frac{d^2y}{dx^2} if,\ y + siny = x$ [A] [4]

14 Find $\frac{dr}{ds}, if\ r = \frac{1}{3s^2} - \frac{5}{2s}$ [A] [4]

15 Evaluate the limit $\lim_{x \to 0} \frac{\sin x}{\tan x}$ without using L'Hôpital's rule and then verify that your answer is correct using L'Hôpital's rule. [A] [6]

16 Find $\lim_{x \to +\infty} \frac{x^2}{e^{3x}}$ [A] [6]

(5 × 6 = 30)

Part C
Essay Type Questions
Answer any three questions. Each question carries 10 marks

17 Let p, q, r be propositions [U] [1]

p: You have flu.

q: You miss the final examination

r: You pass the course

Express each of these expressions as an English sentence

i. $p \rightarrow q$

ii. $\neg q \leftrightarrow r$

iii. $q \rightarrow \neg r$

iv. $p \vee q \vee r$

v. $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

18 a) Find the composites and their domains where f(x) = x + 5 and [U] [2]
$g(x) = x^2 - 3$

(i) $f \circ g$

(ii) $g \circ f$

(iii) $f \circ f$

(iv) $g \circ g$

b) Given the function $y = \sqrt{x}$.

Find the formulae to

(i) stretch the graph $f \circ f$ horizontally by a factor of 3

(ii) compress the graph vertically by a factor of 3

19 (a) Find the inverse of the function f(x) = 2x + 5. [An] [3]

(b) Verify that your answer is correct by composing f and f^{-1}.

(c) Determine the domain and range of $f^{-1}(x)$.

20 Evaluate y'(x) for the following functions: [A] [4]

a) $y = \dfrac{x^3 + 2x^2 - 1}{x + 5}$

b) $y = \dfrac{x^2 - 1}{x^4 + 1}$

21 Calculate the relative maximum and minimum of $sin(2x)\ for\ 0 < x < \pi$ [U] [5]

22 Given the function $f(x) = x^3 - 3x^2 + 3x - 100$ [U] [5]

(a)Find the intervals in which the function is concave up.

(b)Find the intervals in which the function is concave down.

(c)Find the inflection points of f.

(3 × 10 = 30)

END OF THE QUESTION PAPER

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