QP CODE: 25010008

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) SPECIAL REAPPEARANCE EXAMINATIONS, FEBRUARY 2025

Fifth Semester

CORE COURSE - MM5CRT03 - ABSTRACT ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2022 Admission Only

CCDF9CD1

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Check whether usual multiplication is a binary operation on the set $\mathbb{C}$.
2. Write two examples for non structural property of a binary structure $\langle S, * \rangle$.
3. Define order of a group.
4. Define the **symmetric group on** n **letters**. What is its order?
5. Define the **left regular representation** of a group G.
6. Define **even** and **odd** permutations. Give examples.
7. Define the **alternating group** A_n **on** n **letters**. What is its order?
8. Check whether $f : (\mathbb{R}, +) \rightarrow (\mathbb{Z}, +)$ defined by $f(x) = \lfloor x \rfloor$, the greatest integer $\leq x$ is a group homomorphism or not.
9. Show that S_n is not a simple group when n$\geq$ 3.
10. Define a) a commutative ring b) a ring with unity
11. Let R be a commutative ring with unity of characteristic 4 . Compute and simplify $(\mathrm{a} + \mathrm{b})^4$ for $a, b \in R$
12. Mark each of the following true or false.
    a) A ring homomorphism $\phi : R \rightarrow R'$ carries ideals of R into ideals of R'
    b) A ring homomorphism is one to one if and only if the kernel is { 0 }

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that $\langle \mathbb{Q}^+, * \rangle$ is a group, where $*$ is defined by $a * b = ab/2$.

14. Find the quotient q and remainder r when 38 is divided by 7 according to the division algorithm.

15. Find all orders of subgroups of the group $\mathbb{Z}_6$.

16. Prove that every permutation σ of a finite set is a product of disjoint cycles. Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$ in S_8 as a product of transpositions.

17. State and prove the **theorem of Lagrange**.

18. Show that composition of group homomorphisms is again a group homomorphism.

19. Prove that the factor group of cyclic group is cyclic.

20. Prove that $M_n(R)$ is a ring where $M_n(R)$ is the collection of all n x n matrices having elements of R as entries

21. Let N be an ideal of a ring R . Prove that $\gamma : R \to R/N$ given by $\gamma(x) = x + N$ is a ring homomorphism with kernel N.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Let G be a group with binary operation $*$ .Then prove the following:
    a) The left and right cancellation laws hold in G .
    b) The linear equations $a * x = b$ and $y * a = b$ have unique solutions x and y in G, where a and b are any elements of G .

23.
    1. Let H be a subgroup of a group G. Let the relation $\sim_R$ be defined on G by $a \sim_R b$ if and only if $ab^{-1} \in H$. Then show that $\sim_R$ is an equivalence relation on G. What is the cell in the corresponding partition of G containing $a \in G$?
    2. Let H be the subgroup $< \mu_1 >= \{\rho_0, \mu_1\}$ of S_3. Find the partitions of S_3 into left cosets of H, and the partition into right cosets of H.

24. Let H be a subgroup of a group G. prove that $aHbH = abH$ defines a binary operation on G/H if and only if H is a normal subgroup of G. Then furthere show that if H is a normal subgroup of a group G then G/H is a group. under the binary operation $aHbH = abH$.

25. a) Prove that the divisors of 0 in Zn are those nonzero elements that are not relatively prime to n .
    b) Find the divisors of Z_{16}
    c) Prove that Zp , where p is prime has no divisors of 0.

(2×15=30)