25010002

QP CODE: 25010002

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) SPECIAL REAPPEARANCE EXAMINATIONS, FEBRUARY 2025

Fifth Semester

CORE COURSE - MM5CRT01 - MATHEMATICAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2022 Admission Only

891EB84A

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Exhibit a bijection between N the set of natural numbers and the set of all odd numbers greater than 12.
2. Define absolute value function.
3. Prove that $a \leq b$ for all $a \in A, b \in B$ imply that $SupA \leq SupB$.
4. If $t > 0$ prove that there exist an $n_t \in N$ such that $0 < \dfrac{1}{n_t} < t$.
5. Prove that $lim(\frac{1}{n}) = 0$.
6. Prove that $lim(\frac{2n}{n^2+1}) = 0$.
7. Write a short note on Euler number.
8. Give an example of an unbounded sequence that has a convergent subsequence. Explain.
9. Let (x_n) and (y_n) be two sequences of positive real numbers and suppose that for some positive real number L, $lim(\frac{x_n}{y_n}) = L$, then prove that lim x_n = $+\infty$ if and only if lim y_n = $+\infty$.
10. If a series in $\mathbb{R}$ is absolutely convergent, then it is convergent.

11. Define an Alternating Series. Give an example.

12. True or False " The limit of a function f exist at a point c only if f is defined at c". Give justifications.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. State and prove characterisation of intervals theorem.

14. Prove that $x \in [0,1]$ then the binary representation of x forms a sequnce consisting only $0, 1$?

15. If c > 0, prove that lim $(c^{1/n}) = 1$.

16. Let X = (x_n) and Y = (y_n) be sequences of real numbers that converges to x and y respectively. Prove that the sequences X+Y converges to x+y.

17. Let Y = (y_n) be the sequence defined as $y_1 = 1$ and $y_{n+1} = \frac{2y_n+3}{4}, n \geq 1.$ Prove that lim Y = $\frac{3}{2}$.

18. State and prove comparison test for real sequences.

19. Test the convergence and absolute convergence of the series whose nth term is $\frac{n^n}{(n+1)^{n+1}}$.

20. Check whether the one-sided limits of the function $g(x) = e^{\frac{1}{x}}$ at $x = 0$ exist or not.

21. Give an example of a function that has a right-hand limit but not a left-hand limit at a point.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove that the set of all real numbers is a complete ordered field.

23. (a) State and prove Cauchy Convergence Criterion.
(b) Let Y = (y_n) be th esequence of real numbers defined as $y_1 = 1$, $y_2 = \frac{1}{1!} - \frac{1}{2!}$,..., $y_n = \frac{1}{1!} - \frac{1}{2!} + \ldots + \frac{(-1)^{n+1}}{n!}$. Prove that sequence Y is convergent.

24. State and prove Raabe's test. Use this test to study the convergence of $\sum_1^{\infty} \left(\frac{n}{(n^2+1)} \right)$.

25. (a) Let $f : A \to \mathscr{R}$ and c be a cluster point of A. Then prove the following are equivalent.

- $\lim\limits_{x \to c} f = L$.
- For every sequence (x_n) in A that converges to c such that $x_n \neq c$ for all $n \in \mathscr{N}$,the sequence $(f(x_n))$ converges to L.

(b) Show that $\lim\limits_{x \to 0} sgn(x)$ does not exist , Where sgn is the $signum$ function.

(2×15=30)