QP CODE: 25022248

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Reg No : ....

Name : ....

M.Sc DEGREE (CSS) SPECIAL REAPPEARANCE EXAMINATION, APRIL 2025

Third Semester

CORE - ME010303 - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

M.Sc MATHEMATICS , M.Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

0F375A56

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the Fourier Series for $f(x) = 6x, 0 < x < 2\pi$
2. Show by an example that Lebesgue integrability of f and g alone will not give a convolution integral of f and g.
3. Define directional derivative. Find the directional derivative of $f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4}; x \neq 0 \\ 0\ ;\ x = 0 \end{cases}$ at $\mathbf{0}$.
4. Find the Jacobian matrix of $\mathbf{f} : \mathbf{R}^2 \to \mathbf{R}^2$ defined by $\mathbf{f}(x,y) = (x^2y, 5x + siny)$.
5. Show that the mean value theorem for functions from $\mathbf{R}$ to $\mathbf{R}$ does not hold for the function $\mathbf{f} : \mathbf{R} \to \mathbf{R}^2$ defined by the equation, $\mathbf{f}(t) = (cost, sint)$.
6. Find the Jacobian determinant for the function $f(z) = e^z$
7. For some integer $n \geq 1$, let f have a continuous n^th derivative in the open interval (a, b) . Also given that for some interior point c in (a, b) , $f'(c) = f''(c) = ... = f^{n-1}(c) = 0$ but $f^n(c) \neq 0$. If n is even and $f^n(c) > 0$ then prove that f has a local minimum at c.
8. Prove that the function $f : R^2 \to R$ defined by $f(x,y) = (y - x^2)(y - 2x^2)$does not have a local maximum or local minimum at $(0, 0)$
9. $Define\ projection\ functions\ in\ R^n.$
10. $Define\ k\ forms.\ Write\ standard\ presentation\ of$ $\omega = x_2dx_2 \wedge dx_1 - x_2dx_3 \wedge dx_2 - x_1dx_2 \wedge dx_3 + x_1dx_1 \wedge dx_2$

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that every real valued and continuous function on a compact interval can be uniformly approximated by a polynomial.

12. If$p > 0, q > 0$, prove that the beta function can be expressed using gamma function as $\mathcal{B}(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$. Evaluate $\mathcal{B}(4,2)$

13. Define differentiability of a function $\mathbf{f} : S \to \mathbf{R}^m (S \subseteq \mathbf{R}^n)$ at a point $\mathbf{c} \in S$. Show that if $\mathbf{f}$ is differentiable at $\mathbf{c} \in S$ then all directional derivatives exist at that point. Also show that the differentiability of a function at a point implies continuity at that point.

14. State and prove a sufficient condition for the complex-valued function of a complex variable $f = u + iv$ to have a derivative at an interior point $c \in S \subseteq \mathbf{C}$.

15. (a) Let A be an open subset of R^n and assume that $f : A \to R^n$ is continuous and has finite partial derivatives $D_j f_i$ on A. If f is one-to-one on A and if $J_f(x) \neq 0$ for each x in A then prove that $f(A)$ is open.

    (b) Let A be an open subset of R^n and assume that $f : A \to R^n$ has continuous partial derivatives $D_j f_i$ on A. If $J_f(x) \neq 0$ for all x in A then prove that f is an open mapping .

16. (a) Define Quadratic form. When will you say that a quadratic form is positive definite.

    (b) Find the saddle point of the function $f(x,y) = 4xy - x^4 - y^4$

17. $Define\ integral\ of\ f\ over\ I^k.\ Show\ that\ the\ k\ integration\ of\ f\ does\ not\ depend\ on\ the\ order.$

18. $If\ \Phi(r,\theta,\phi) = (x,y,z)\ where\ x = rsin\theta cos\phi,\ y = rsin\theta sin\phi,\ z = rcos\theta\ and\ D\ is\ the\ 3-cell\ defined\ by\ 0 \leq r \leq 1,\ 0 \leq \theta \leq \pi,\ 0 \leq \phi \leq 2\pi,\ then\ find\ \int_{\Phi} dx \wedge dy \wedge dz.$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. State and prove the Exponential form of Fourier Integral Theorem.

20. State and prove the chain rule.

21. Assume that one of the partial derivatives $D_1 f, D_2 f, \ldots . D_n f$ exist at c and the remaining $n-1$ partial derivatives exist in some n-ball $B(c)$ and are continuous at c . Prove that f is differentiable at c

22. $Suppose\ F\ is\ a\ C^{'}\ mapping\ of\ an\ open\ set\ E \subset R^n\ into\ R^n,\ 0 \in E,\ F(0) = 0\ and\ F^{'}(0)\ is\ invertible.\ Then\ there\ is\ a\ nbd\ of\ 0\ inR^n\ in\ which\ F(x) = B_1 B_2 \ldots . B_{n-1} G_n \circ \ldots . \circ_1 G(x).$

(2×5=10 weightage)