QP CODE: 25022246

25022246

Reg No : ....

Name : ....

M.Sc DEGREE (CSS) SPECIAL REAPPEARANCE EXAMINATION, APRIL 2025

Third Semester

CORE - ME010302 - PARTIAL DIFFERENTIAL EQUATIONS

M.Sc MATHEMATICS , M.Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

6B2C2215

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. *Verify that the equation* $(x^2z - y^3)\,dx + 3xy^2\,dy + x^3\,dz = 0$ *is integrable.*
2. *Eliminate the constants* a *and* b *from the equation* $2z = (ax+y)^2 + b$
3. Find the general integral of the partial differential equation $(2xy-1)p + (z-2x^2)q = 2(x-yz)$.
4. Verify that the equation $z = \sqrt{(2x+a)} + \sqrt{2y+b}$ is a complete integral of the partial differential equation $z = \frac{1}{p} + \frac{1}{q}$.
5. Prove that the equations $p = P(x,y),\ q = Q(x,y)$ are compatible if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$.
6. Find the complementary function of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$.
7. Show that $F(D, D')e^{ax+by}\phi(x,y) = e^{ax+by}F(D+a, D'+b)\phi(x,y)$.
8. Find the particular integral of $[D^2 - {D'}^2]z = x - y$.
9. Write Monge's equations.
10. Prove that the function $\phi = sinxcoshy + 2cosxsinhy + x^2 - y^2 + 4xy$ is Harmonic.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Find the integral curves of* $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$

Turn Over

12. *Show that the orthogonal trajectories on the hyperboloid $x^2+y^2-z^2=1$ of the conics in which it is cut by the system of planes $x+y=c$ are its curves of intersection with the surfaces $(x-y)z=k$, where k is a parameter.*

13. Find the complete integral of the equation $2(y+zq)=q(xp+yq)$.

14. Show that the integral surface of the equation $2y(1+p^2)=pq$ which is circumscribed about the cone $x^2+z^2=y^2$ has the equation $z^2=y^2(4y^2+4x+1)$.

15. By Jacobi's method solve $p^2x+q^2y=z$.

16. Verify that the PDE $z_{xx}-\frac{1}{x}z_x=4x^2z_{yy}$ is satisfied by $z=f(x^2-y)+g(x^2+y)$.

17. Describe the method of seperation of variables for solving a second order linear partial differential equations.

18. Show that in cylindrical coordinates ρ, z, ϕ , the Laplace's equation has solutions of the form $R(\rho)exp(\pm mz \pm in\phi)$ where $R(\rho)$ is a solution of Bessel's equation $\frac{d^2R}{d\rho^2}+\frac{1}{\rho}\frac{dR}{d\rho}+(m^2-\frac{n^2}{\rho^2})R=0$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Prove that necessary and sufficient condition that the Pfaffian differential equation $X.dr=0$ should be integrable is that $X.curlX=0$.
b) Prove that given one integrating factor of the Pfaffian differential equation $X_1dx_1+X_2dx_2+\cdots+X_ndx_n=0$, we can find an infinity of them.

20. a) If $u_i(x_1,x_2,\ldots,x_n,z)=c_i,(i=1,2,\ldots,n)$ are the independent solutions of the equations $\frac{dx_1}{P_1}=\frac{dx_2}{P_2}=\ldots=\frac{dx_n}{P_n}=\frac{dz}{R}$, then prove that the relation $\Phi(u_1,u_2,\ldots,u_n)=0$, in which Φ is arbitrary, ia a general solution of the linear partial differential equation $P_1\frac{\partial z}{\partial x_1}+P_1\frac{\partial z}{\partial x_2}+\ldots+P_1\frac{\partial z}{\partial x_n}=R$.
b) Find the general solution of the partial differential equation $(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$ if u is a function of x, y and z.

21. Reduce the equation to canonical form and solve $y^2u_{xx}-2xy\,u_{xy}+x^2u_{yy}=\frac{y^2}{x}u_x+\frac{x^2}{y}u_y$.

22. (a) Prove that if $f(x,y,z)=c$ is a family of equipotential surfaces, then $\frac{\nabla^2 f}{|\nabla f|^2}$ is a function of f alone.
(b) Show that the right circular cones $x^2+y^2=cz^2$ forms a set of equipotential surfaces and show that the corresponding potential function is of the form $Alogtan\frac{\theta}{2}+B$ where are constants and θ is the usual polar angle.

(2×5=10 weightage)