QP CODE: 25022244

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Reg No : ....

Name : ....

M.Sc DEGREE (CSS) SPECIAL REAPPEARANCE EXAMINATION, APRIL 2025

Third Semester

CORE - ME010301 - ADVANCED COMPLEX ANALYSIS

M.Sc MATHEMATICS , M.Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

C6FAB2FB

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Prove that $logr$ is a harmonic function.
2. State Schwarz's theorem.
3. Write the Taylor series expansion of arc tanz about the origin.
4. Expand $f(z) = \frac{1}{z(z-1)}$ as Laurent's series in powers of z.
5. State Mittag-Leffler's theorem.
6. *Prove that* $\zeta(s)\Gamma(s) = \int_0^\infty \frac{x^{s-1}}{e^x-1}dx$ for $\sigma = Res > 1$.
7. *Prove that a sequence of functions in $\mathcal{F}$ converges uniformly to f on compact subsets if and only if it converges to f with respect to the distance function ρ in $\mathcal{F}$.*
8. State Arzela- Ascoli's theorem.
9. What is meant by the boundary behavior?
10. Prove that $\wp(2z) = \frac{1}{4}[\frac{\wp''(z)}{\wp'(z)}]^2 - 2\wp(z)$

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that if $f(z)$ is analytic in a region Ω, then $\overline{f(\bar{z})}$ is analytic in $\Omega^* = \{\bar{z}; z \in \Omega\}$.

12. State and prove *Harnack's* inequality.

13. Prove that a necessary and sufficient condition for the absolute convergence of the product $\Pi_1^\infty (1 + a_n)$ is the convergence of the series $\Sigma_1^\infty |a_n|$.

14. Obtain a formula for $\Gamma(z)\Gamma(1-z)$. Deduce $\Gamma(\frac{1}{2})$.

15. *Prove that* $\frac{1}{\zeta(s)} = \prod_{n=1}^{\infty}(1 - p_n^{-s})$ *where* $p_1, p_2, \ldots$ *are ascending sequence of primes and* $\sigma =$ *Re* $s > 1$.

16. *Prove* $\xi(s)$ *is entire and* $\xi(s) = \xi(1-s)$ *where* $\xi(s) = \frac{1}{2}s(1-s)\pi^{-(\frac{s}{2})}\Gamma(\frac{s}{2})\zeta(s)$.

17. Define the Riemann mapping. Let Ω be a simply connected region other than the complex plane. Prove that the Riemann mapping from Ω to the unit disk is unique.

18. Prove that an elliptic function cannot have a single simple pole.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Define a subharmonic function. State and prove any three properties.

20. Obtain Jensen's formula. Deduce Poisson-Jensen formula.

21. (i) Prove that $\zeta(s)$ has no zeros in the half plane $\sigma > 1$.
    (ii) Differentiate between the trivial and non trivial zeros of the Zeta function.

22. (a)Define Weirstrass' $\wp$ -function .
    (b) Exoress Weirstrass' $\wp$ -function as a Laurent series about the origin.
    (c) Prove that $\wp(z) = \frac{1}{z^2} + \sum_{\omega \neq 0}(\frac{1}{(z-\omega)^2} - \frac{1}{\omega^2}$.

(2×5=10 weightage)