QP CODE: 24803037

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Reg No : ....

Name : ....

INTEGRATED MSC DEGREE EXAMINATION, MAY 2024

Seventh Semester

INTEGRATED MSC BASIC SCIENCE-STATISTICS

CORE - IST7CR01 - PROBABILITY THEORY

2020 Admission Onwards

FE6ADD36

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find $\lim_{n\to\infty} A_n$ if $A_n = (0, 1 + \frac{1}{n}), n = 1, 2, \ldots$
2. Define minimal sigma field.
3. Define monotone class.
4. A box contains good and defective items. If an item drawn is good, we assign the number 1 to the drawing, otherwise, the number 0. Let p be the probability of drawing at random a good item. Then find $F(x)$.
5. State Jordan decomposition theorem.
6. If the moment of order t exists for a random variable X, then show that moments of order $0 < s < t$ exist.
7. Let $X_n \xrightarrow{r} X$ for some $r > 0$. Then show that $X_n \xrightarrow{P} X$
8. Define weak convergence of distribution functions.
9. State Kolmogorov strong law of large numbers for iid random variables.
10. State Liapounov's central limit theorem.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Show that $\mathscr{A}$ is a field iff it is closed under complementation and finite intersection.
12. Differentiate between Lesbesgue measure and Lesbesgue Stieltjes measure.

13. In answering a question on a multiple choice test, a candidate either knows the answer with probability $p\,(0 \leq p \leq 1)$ or does not knows the answer with probability $1-p$. If he knews the answer, he puts down the correct answer with probability 0.99, whereas if he guesses, the probability of his putting down the correct result is $\frac{1}{k}$ (k choices to the answer). Find the conditional probability that the candidate knew the answer to a question given that he has made the correct answer. Show that this probability tends to 1 as $k \to \infty$

14. If A and B are independent events then show that a) A^c and B are independent b) A^c and B^c are independent.

15. Check whether $E(X)$ exists for the Cauchy pdf $f(x) = \frac{1}{\pi}\frac{1}{1+x^2},\ -\infty < x < \infty$

16. State and prove Jensen's inequality.

17. State and prove weak law of large numbers.

18. Determine whether the weak law of large numbers holds for the following sequence $\{X_n\}$ of independent random variables with $P[X_n = 2^n] = \frac{1}{2} = P[X_n = -2^n]$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. State and prove inclusion exclusion principle.

20. A die is tossed two times. Let X be the sum of face values on the two tosses and Y be the absolute value of the difference in face values. What is Ω? What values do X and Y assign to points of Ω? Check whether X and Y are random variables.

21. a) Let $\{X_n\}$ be a sequence of random variables such that $X_n \xrightarrow{L} X$ and let C be a constant. Then show that i) $X_n + C \xrightarrow{L} X + C$ and ii) $CX_n \xrightarrow{L} CX, C \neq 0$
b) Let $X_n, n = 1, 2, \ldots$ and X be continuous random variables such that $f_n(x) \to f(x)$ for (almost) all x as $n \to \infty$. Here f_n and f are the pdf's of X_n and X respectively. Then show that $X_n \xrightarrow{L} X$

22. a) 30 unbiased dice are thrown. Find the approximate probability that the sum of the numbers shown is between 90 and 120 using CLT.
b) 30 real numbers are randomly selected from the interval (0,2). Find the approximate probability that the sum of these numbers is greater than 25.

(2×5=10 weightage)