QP CODE: 24001057

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, MARCH 2024

Sixth Semester

CORE COURSE - MM6CRT03 - COMPLEX ANALYSIS

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

A66A3E10

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define Interior point and Boundary point in terms of neighbourhood
2. Find $f'(z)$ where $f(z)=\frac{z-1}{2z+1}$ where $z\neq \frac{-1}{2}$
3. If u+iv is analytic, then under what condition will v+iu be analytic
4. Find i^i and its principal value
5. Separate the real and imaginary parts of Sinhz.
6. Define Simple closed curve.
7. What is the value of $\int_C (z-1)dz$ where C is the line segment z=x, $0 \leq x \leq 2.$
8. Define simply connected and multiply connected domain.
9. Define the limit of an infinite sequence of complex numbers.
10. With the aid of the identity $\cos z = -\sin(z-\frac{\pi}{2})$, expand $\cos z$ into a Taylor series about the point $z_0 = \frac{\pi}{2}$
11. State Cauchy's Residue Theorem.
12. Prove that if the improper integral over $-\infty < x < \infty$ exists, then its Cauchy Principal Value exists.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. If a function is analytic,Show that it is independent of $\bar{z}$

14. Prove that |exp(-2z)|<1 if and only if Re(z)>0

15. Find where $\tan^{-1} z = \frac{i}{2} \log \frac{i+z}{i-z}$ is analytic

16. Evaluate $\int_C \frac{1}{z^2+2z+2} dz$ where C is the circle |z|=1.

17. Evaluate $\displaystyle\int_C \frac{\sinh z}{(2z-z^2)^2}$ Where C is the circle $|z| = 1$ oriented counterclockwise

18. State and prove Cauchy's inequality.

19. Use Maclaurin's series expansion of $\sin z$ to obtain such a series for $\cos z$

20. Using residues, evaluate $\displaystyle\int_C e^{\left(\frac{1}{z^2}\right)} dz$ where C is the unit circle about the origin.

21. State the characterization of poles of order m of a complex function $f(z)$ and the formula for residue at z_0 of the poles of order m. Find the residue at $z = i$ of $f(z) = \frac{z^3+2z}{(z-i)^3}$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) State and prove the sufficient condition for a function f(z) to be differentiable.
b) Show that the function f(z) =ln(|z|)+i Arg(z) is analytic onits domain of definition and f'(z)=$\frac{1}{z}$

23.
- Prove that any polynomial of degree n has atleast one zero
- State and Prove Liouvilles theorem

24. a) Derive the Laurent series expansion of $\frac{e^z}{(z+1)^2}$ in terms of $z+1$, if $0 < |z+1| < \infty$
b) Let $f(z) = \frac{1}{(z-i)^2}$. Use Laurent series expansion to prove that
$$\int_C \frac{dz}{(z-i)^{-n+3}} = 2\pi i, n = 2$$
c) Show that for $0 < |z-1| < 2$ $\displaystyle\frac{z}{(z-1)(z-2)} = \frac{-1}{2(z-1)} - 3\sum_{n=0}^{\infty} \frac{(z-1)^n}{2^{n+1}}$

25. Define the Removable singular points, essential singular points and a pole of order m, of a complex function with examples. Verify the examples with their series representations.

(2×15=30)