25019347

QP CODE: 25019347

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS)) REGULAR/ IMPROVEMENT/ REAPPEARANCE / MERCY CHANCE EXAMINATIONS, FEBRUARY 2025

Fourth Semester

Complementary Course - MM4CMT01 - MATHEMATICS - FOURIER SERIES, LAPLACE TRANSFORM AND COMPLEX ANALYSIS

(Common for B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III, B.Sc Geology and Water Management Model III, B.Sc Geology Model I, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance)

2017 Admission Onwards

F0354673

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define Fourier Cosine Series?
2. Define power series and its center. If the centere is zero, write the power series?
3. Find $\mathscr{L}(1)$
4. Using linearlty propert of Laplace transform find $\mathscr{L}(sinh\ at)$
5. Write a relation between $\mathscr{L}\{t^n f(t)\}$ and $\mathscr{L}\{f(t)\}$
6. Find the real and imaginary parts of $z_1 z_2$ where $z_1 = 8 - 3i$ and $z_2 = 9 + 2i$.
7. Find the real and imaginary parts of $\frac{1}{z}$ where $z = 4 - 5i$.
8. Find $|e^z|$, where $z = 2 + 3\pi i$.
9. Evaluate $(-3)^{3-i}$.
10. Find the centre and radius of the circle |z - 4 + 2i| =3.
11. State true or false: If a complex function f is analytic at a point, then its derivatives of all orders are also analytic at that point.
12. State Liouville's Theorem.

(10×2=20)

Part B

Answer any ***six*** *questions.*

Each question carries ***5*** *marks.*

13. Find the Fourier series expansion of $f(x) = \begin{cases} \frac{\pi}{2} + x & -\pi < x < 0 \\ \frac{\pi}{2} - x & 0 < x < \pi \end{cases}$ with $f(x + 2\pi) = f(x)$

14. Find the Fourier cosine series of $f(x) = \sin x, x \in [0, \pi]$

15. Evaluate $\mathscr{L}^{-1}(\frac{4s+1}{s^2-16})$

16. Solve the initial value problem $y'' + 2y' + y = e^{-t}$ with $y(0) = -1$ and $y'(0) = 1$, using Laplace transforms.

17. Show that $|z_1 + z_2| \leq 3$, where $z_1 = \frac{1}{2} - \frac{\sqrt{3}}{2}i$ and $z_2 = \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}i.$

18. Solve the equation $z^4 + 4 = 0.$

19. Check the analyticity of $(z - 2)^3$.

20. Apply Cauchy's integral theorem to show that $\int_C \frac{z^2}{z-3} dz = 0$, C is the circle |z| = 1.

21. Evaluate $\oint_C \frac{z^2-1}{z^2+1} dz$ using Cauchy's integral formula, C is the circle |z - i| = 1.

(6×5=30)

Part C

Answer any ***two*** *questions.*

Each question carries ***15*** *marks.*

22. Derive the Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}[(x^2 - 1)^n]$ and find the expressions of Legendre polynomials upto degree 5 from this formula

23. Evaluate (a) $\mathscr{L}^{-1}\{\frac{1}{s^3-4s}\}$ (b) $\mathscr{L}^{-1}\{\frac{s-a}{s(s+a)}\}$ (c) $\mathscr{L}^{-1}\{\frac{1}{s^4-2s^3}\}$

24. Verify that $u = x^2 - y^2 - y$ is harmonic or not in the entire complex plane and find a conjugate harmonic function v of u. Also find the corresponding analytic function $f(z)$.

25. Evaluate $\int_C f(z)dz$ where f(z) = x^2 + 3ixy and C is the line segment joining 1+ i to 2 - i.

(2×15=30)