**QP CODE: 25020815**

25020815

**Reg No** : .....................

**Name** : .....................

# B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE / MERCY CHANCE EXAMINATIONS, FEBRUARY 2025

## Sixth Semester

## CORE COURSE - MM6CRT03 - COMPLEX ANALYSIS

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

2B5905A5

Time: 3 Hours Max. Marks : 80

### Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define Interior point and Boundary point in terms of neighbourhood.
2. Examine the differentiability of function $f(z)=|z|^2$
3. Solve the equation $e^z=1+i$
4. Separate the real and imaginary parts of Sinhz.
5. Evaluate $\cosh^{-1}(-1)$
6. Define Simple closed curve.
7. Define simply connected and multiply connected domain.
8. State Liouville's theorem.
9. $Evaluate \quad lim_{n\to\infty} z_n \; where z_n = \frac{-2+i(-1)^n}{n^2}$
10. $Use\ Laurent\ series\ expansion\ to\ show\ that$ $\int_C e^{\frac{1}{z}}\,dz = 2\pi i$ $where\ C\ is\ any\ positively\ oriented\ simple\ closed\ contour\ around\ origin.$
11. State Cauchy's Residue Theorem.
12. State a sufficient condition for an isolated singular point $z_0$ of a function $f(z)$ to be a pole of order $m$. Also give the formula for the residue at $z_0$

(10×2=20)

### Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Find an analytic function whose real part is $e^x$(x cosy-y siny) and which takes the value e at z=1.

14. Find an analytic function f(z) in terms of z and with real part $u = y - \frac{1}{2}y^2 + \frac{1}{2}x^2$

15. Show that $Re[\log(z-1)] = \frac{1}{2}\ln[(x-1)^2 + y^2], z \neq 1.$

16. Evaluate $\int_C \frac{1}{z^2+2z+2}dz$ where C is the circle |z|=1.

17. Let a function f be analytic everywhere within and on a closed countour C,taken in the positive sense .If $z_0$is any point interior to C,Prove that $\int_C \frac{f(z)}{z-z_0}dz = 2\pi i f(z_0).$

18. Prove that a function f is analytic at a given point, then its derivative of all orders are analytic at that point.

19. *Expand* $f(z) = \frac{1+2z^2}{z^3+z^5}$ *in powers of z and specify the domain in which the expansion is valid.*

20. Using residues, evaluate $\int_C e^{\left(\frac{1}{z^2}\right)}dz$ where C is the unit circle about the origin.

21. Prove that if the improper integral of $f(x)$over $-\infty < x < \infty$ exists, then its Cauchy Principal Value exists. Is the converse true? Justify your answer.

(6×5=30)

**Part C**

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) State and prove the sufficient condition for a function f(z) to be differentiable.
b) Show that the function f(z) =ln(|z|)+i Arg(z) is analytic onits domain of definition and $f'(z)=\frac{1}{z}$

23. Evaluate $\int_c f(z)dz,$ where $f(z) = \exp(\pi\bar{z})$ and C is the boundary of the square with vertices at the points 0, 1, 1+i and i, the orientation of C being in the counter clockwise direction.

24. *Prove that a function* $f(z)$ *which is analytic through out* $|z - z_0| < R_0$ *has a Taylor series representation about* $z = z_0$ *of the form* $\sum_{n=0}^{\infty} a_n(z-z_0)^n.$

25. Define the Removable singular points, essential singular points and a pole of order $m$ , of a complex function with examples. Verify the examples with their series representations.

(2×15=30)