QP CODE: 25020813

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE / MERCY CHANCE EXAMINATIONS, FEBRUARY 2025

Sixth Semester

CORE COURSE - MM6CRT02 - GRAPH THEORY AND METRIC SPACES

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

5E1AFC83

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define a Graph.What is a trivial graph?
2. Give two different drawings of K_4 which are isomorphic?
3. Draw all non-isomorphic complete bipartite graphs with atmost 4 vertices.
4. Define an edge deleted subgraph.
5. Define a forest. Draw one example.
6. Define n - connected graph and Internally disjoint paths in a graph.
7. Define Euler trail and Euler tour of graph G.
8. Define Hamiltonian graph. Give an example.
9. Define an open set in a metric space X.
10. Define closure of a set A. If A = (0,1), What is closure of A?
11. Define convergence of a sequence in a metric space.
12. State Baire's Theorem.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Define eccentricity, diameter and radius of a connected graph G with vertex set V. Find the radius and diameter of Petersen graph?
14. Define adjacency matrix of a graph. Find the graph whose adjacency matrix is $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. What can you say about the graph if all the entries of the main diagonal are zero?

15. Prove that a graph G is connected if and only if it has a spanning tree.

16. a) Define cut vertex of a graph.

b) Let v be a vertex of the connected graph G . Then prove that 'v' is cut vertex of G if and only if there are two vertices 'u' and 'w' of G , both different from 'v', such that 'v' is on every u – w path in G.

17. Prove that a simple graph G is Hamiltonian if and only if its closure C(G) is Hamiltonian.

18. Show that d: R× R→R define by d(x,y) = |x-y|, for every x,y in R is a metric on R.

19. Is int(A∪B) = int A ∪ int B? Justify your answer.

20. If a convergent sequence in a metric space has infinitely many distinct points, then prove that its limit is a limit point of the set of points of the sequence.

21. Let X and Y be metric spaces and a mapping of X into Y. Prove that f is continuous if and only if $x_n \to x$ implies $f(x_n) \to f(x)$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a)State and prove First theorem of graph theory.
(b)Prove that in any graph G there is an even number of odd vertices.
(c)Prove that it is impossible to have a group of nine people at a party such that each one knows exactly five of the others in the group.

23. a) Let 'e' be an edge of the graph G and let 'G – e' be the sub graph obtained by deleting e. Then prove that $\omega(G) \leq \omega(G-e) \leq \omega(G)+1$.
b) Prove that an edge 'e' of a graph G is a bridge if and only if 'e' is not a part of any cycle in G.

24. a) Prove that in a metric space X, every closed sphere is a closed set.
b) Prove that A is closed if and only if $\bar{A} = A$.
c) Prove that in a metric space X, any intersection of closed sets in X is closed.

25. (a) Let X be a complete metric space and let Y be a subspace of X. Prove that Y is complete if and only if it is closed.
(b) State and prove Cantor's Intersection Theorem.

(2×15=30)