QP CODE: 25020810

25020810

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE / MERCY CHANCE EXAMINATIONS, FEBRUARY 2025

Sixth Semester

CORE COURSE - MM6CRT01 - REAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

56043023

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Give an example of functions f and g that are both discontinuous at a point c in R such that the sum f+g is continuous at c.
2. Show that the continuous image of an open interval need not be an open interval.
3. Define Monotone function.Show that such functions need not be continuous.
4. Define the derivative of a function f at a point c .
5. Using the chain rule, find the derivative of $f^n(x)$, where $f : I \to R$ is differentiable and $n \geq 2, n \in N$.
6. Find $\lim_{x\to 0} \dfrac{e^x - 1}{x}$.
7. Find the norm of the partition $\mathcal{P} = \{1, 1.2, 1.65, 2, 2.31, 2.72, 3\}$ of the interval $[1, 3]$.
8. Under what circumstances differentiation and Riemann ntegration are inverse to each other.
9. State any theorem which characterises Riemann Integrable function on an interval $[a, b]$.
10. Evaluate $lim(\frac{nx}{1+nx})$ for $x \epsilon R; x \geq 0$.
11. Evaluate $lim(e^{-nx})$ for $x \in R, x \geq 0$.
12. Give an example of a sequence of continuous functions that converges pointwise to a discontinuous limit.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Define Dirichlet's function. Show that it is not continuous at any point of R.

14. Let $a < b < c$. Suppose that f is continuous on [a,b], that g is continuous on [b,c], and that f(b) =g(b). Define h on [a,c] by h(x) = f(x) for $x \in [a, b]$ and h(x) = g(x) for $x \in [b, c]$. Prove that h is continuous on [a,c] .

15. Show that every continuous function on a closed bounded interval I is uniformly continuous on I.

16. Let $f : R \to R$ defined by $f(x) = \begin{cases} x^2, & x \; is \; rational \\ x, & x \; is \; irrational \end{cases}$, Prove that f is differentiable at $x = 0$.

17. State and prove Rolle's theorem.

18. Using Mean value theorem, Prove that $|\sin x - \sin y| \leq |x - y|, \forall x, y \in R$.

19. Suppose that f is continous on on $[a, b]$ and that $f(x) \geq 0$ fo all $x \in [a, b]$ and that $\int_a^b f = 0$. Prove that $f(x) = 0$ for all $x \in [a, b]$.

20. Evaluate $\int_1^4 \frac{cos\sqrt{t}}{\sqrt{t}} dt$.

21. Let $g_n : [0, 1] \to \mathbb{R}$ defined by $g_n(x) = x^n$. Show that (g_n) converges but the limit is not differentiable on $[0, 1]$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a) State and prove Location of Roots Theorem.
    (b) Let $I \subseteq R$ be an interval and let $f : I \to R$ be increasing on I . Suppose that $c \in I$ is not an endpoint of I. Prove that the following statements are equivalent.
    (i) f is continuous at c.
    (ii) $\lim_{x \to c-} f = f(c) = \lim_{x \to c+} f$.
    (iii) $\sup\{f(x) : x \in I, x < c\} = f(c) = \inf\{f(x) : x \in I, x > c\}$.

23. (a.) State and Prove L'Hospital's Rule II
    (b.) Using this, find the following

(i.) $\lim_{x\to 0+} \dfrac{\log \sin x}{\log x}$

(ii.) $\lim_{x\to\infty} \dfrac{\log x}{x}$

24. (a) Suppose that $f:[a,b]\to\mathbb{R}$ and that $f(x)=0$, except for a finite number of ponits $c_1, c_2, \ldots\ldots. c_n$ in $[a,b]$. Prove that $f\in\mathcal{R}[a,b]$ and $\int_a^b f = 0.$

(b) If $g\in\mathcal{R}[a,b]$ and if $f(x)=g(x)$ except for a finite number of ponts in $[a,b]$, prove that $f\in\mathcal{R}[a,b]$ and that $\int_a^b f = \int_a^b g.$

25. (a) State and prove the Cauchy Criterion for Riemann integrability of a function $f:[a,b]\to\mathbb{R}$.

(b) Check the Riemann integrability of Dirichlet function.

(2×15=30)