QP CODE: 25020346

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Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE / MERCY CHANCE EXAMINATIONS, FEBRUARY 2025

Sixth Semester

B.Sc Statistics Model I

CORE COURSE - ST6CRT10 - ANALYTICAL TOOLS FOR STATISTICS

2017 Admission Onwards

26808C07

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that

$$\nabla = 1 - E^{-1}$$

2. Show that

$$D = \frac{1}{h}\left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \dots\right]$$

3. Show that

$$\mu\delta = \frac{1}{2}\left[E - E^{-1}\right]$$

4. Define mean operator μ

5. By constructing a difference table find the sixth term of the sequence 8, 12, 19, 29, 42,….

6. By means of Newtons divided difference formula find a polynomial of the lowest possible degree which takes the values : f(-1) = -21, f(1) = 15,f(2) = 12,f(3) = 3.

7. State Newton's backward interpolation formula.

8. Define Simpson's three-eighth rule.

9. Prove that

$$u = y^3 - 3x^2y + 8$$

is a harmonic function.

10. Show that the function

$$f(z) = e^z$$

is analytic function.

11. State the necessary conditions for f(z) to be analytic.

12. Define upper Riemann integral of a function.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Express the following polynomial in factorial notation and thus find the first three differences

$$f(x) = 7x^4 - 2x^2 + 4$$

14. Use the method of separation of symbols prove that

$$\Delta^n U_{x-n} = U_x - {}_nC_1 U_{x-1} + {}_nC_2 U_{x-2} - \ldots\ldots + (-1)^n U_{x-n}$$

15. Show that n^{th} divided differences of a polynomial of n^{th} degree are constants.

16. Derive Lagrange's interpolation formula

17. By the use of Lagrange's interpolation formula find a polynomial f(x) of degree three f(x) which takes the values

$$f(0) = 1, f(1) = 1, f(2) = 2, f(4) = 5$$

18. Evaluate

$$\int_1^6 \frac{dx}{x}$$

using the Trapezoidal rule take h = 1

19. Prove that

$$u = y^3 - 3x^2 y - 8$$

is a harmonic function. Determine its harmonic conjugate and find the corresponding analytic function

$$f(z) = u + iv$$

20. Find the residue at its poles of the following function

$$f(z) = \frac{9z + i}{z(z^2 + 1)}$$

21. If f is defined on $[0, a]$; $a > 0$ by $f(x) = x^2$ for all $x \in [0, a]$ then $f \in R[0, a]$ and $\int_0^a f(x)dx = \frac{a^3}{3}$

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Derive Gass's Forward interpolation formula. Using the table of values of the function $y = f(x)$, find $f(34)$

x	10	20	30	40	50
f(x)	0.1736	0.3420	0.5	0.6428	0.7660

23. Apply Bessel's formula to find $f(28)$ and $f(31)$, given the following data.

x	10	15	20	25	30	35	40	45
f(x)	4477	5736	7101	8613	10321	12300	14659	17566

24. Evaluate

$$\int_0^6 \sqrt{1 + x + x^2}dx$$

by using Simpson's one third, Simpson's three eighth and Weddle's rule.

25. a) Define the following terms with one example each i) Removable singularity ii) Essential singularity iii) Pole

b) Find the value of the integral

$$\int_C \frac{dz}{z^4(z + 2)}$$

taken counter clockwise around the circle C such that

$$i) \quad C : |z| = 1 \quad ii) \quad C : \quad |z + 1| = 2 \quad iii) \quad C : |z - 2i| = 1$$

(2×15=30)