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Reg. No.....

Name.....

B.C.A. DEGREE (C.B.C.S.S.) EXAMINATION, SEPTEMBER 2024

First Semester

MATRICES, CALCULUS AND LAPLACE TRANSFORMS

(Complementary Mathematics for B.C.A.)

[2013—2016 Admissions]

Time : Three Hours

Maximum Marks : 80

Part A

Answer all questions.

Each question carries 1 mark.

1. What is the rank of a non-singular matrix of order n .
2. Define characteristic polynomial of a square matrix.
3. Find $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x}$.
4. Write quotient rule of differentiation.
5. State Rolle's theorem.
6. Write the order and degree of the equation $\frac{\partial^2 \theta}{\partial x^2} = \frac{\partial \theta}{\partial t}$.
7. Find $\lim_{x \rightarrow 0} \frac{\sin 2x}{5x}$.
8. State linearity property of Laplace transform.
9. Find $L[5t - 3]$.
10. If $L\{F(t)\} = f(s)$, then what is $L[t^n F(t)]$.

(10 × 1 = 10)

Turn over





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Part B

*Answer any **eight** questions.*

Each question carries 2 marks.

11. Define row equivalent canonical matrix of a non-zero matrix.

12. Find the characteristic polynomial of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}$.

13. Reduce to the normal form the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$.

14. If $\lim_{x \rightarrow 4} \frac{f(x) - 5}{x - 2} = 1$. Find $\lim_{x \rightarrow 4} f(x)$.

15. If $f(s) = \frac{\sqrt{s} - 1}{\sqrt{s} + 1}$, find $f'(s)$.

16. Find the absolute maximum and minimum values of $f(x) = \frac{2}{3}x - 5$ in the interval $-2 \leq x \leq 3$.

17. Let $s = \frac{25}{t^2} - \frac{5}{t}$, $1 \leq t \leq 5$ be the position of a body moving on a co-ordinates line for $1 \leq t \leq 5$, with s in meters and t in second. Find the body's displacement and average velocity for the given time interval.

18. Form a partial differential equation by eliminating a and b from $az + b = a^2x + y$.

19. Find the partial differential equation by eliminating the arbitrary function f from $z = f(x^2 - y^2)$.

20. Find the Laplace transform of $\cos at$.





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21. Find $L[t \sin at]$.
22. Find the inverse transform of $\frac{3s+7}{s^2-2s-3}$.

(8 × 2 = 16)

Part C

*Answer any **six** questions.*

Each question carries 4 marks.

23. Find the rank of the matrix $\begin{bmatrix} 1 & 6 & -18 \\ -4 & 0 & 5 \\ -3 & 6 & -13 \end{bmatrix}$ by reducing to Echelon form.
24. Given $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$. Find the inverse of A using Cayley-Hamilton Theorem.
25. Show that the characteristics roots of a real symmetric matrix are real.
26. Use implicit differential to find $\frac{dy}{dx}$ given $x^3 - xy + y^3 = 1$.
27. Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point (0, 2).
28. Consider the function $h(x) = -x^3 + 2x^2$:
- (a) Find the intervals on which $h(x)$ is increasing and decreasing.
 - (b) Identify the function's local extreme values if any.
29. Find the differential equations of all spheres of radius r , having centre in the xy plane.
30. Find the Laplace transform of $e^{-3t} [2 \cos 5t - 3 \sin 5t]$.

Turn over





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31. Evaluate $L \left\{ \int_0^t \frac{e^t \sin t}{t} dt \right\}$.

(6 × 4 = 24)

Part D

*Answer any **two** questions.
Each question carries 15 marks.*

32. (a) Solve the system of equations :

$$5x + 3y + 3z = 48$$

$$2x + 6y - 3z = 18$$

$$8x - 3y + 2z = 21.$$

(b) Determine the eigen values and eigen vectors of the matrix :

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

33. (a) Use chain rule to find the derivative of $(5x^3 - x^4)^7$.

(b) The radius of a circle is changing at the rate of $-2/\pi$ m/s. At what rate is the circle's area changing when $r = 10$ m.

(c) Find the absolute extrema of the function $h(x) = x^{2/3}$ on $[-2, 3]$.

34. Solve the equation $\frac{y-z}{yz} p + \frac{y-x}{zx} q = \frac{x-y}{xy}$.

35. (a) Using convolution theorem find the inverse Laplace transform of $\frac{1}{s^2(s^2+9)}$.

(b) Find $F(t)$ if $L \{F(t)\} = \frac{1}{s^2} = \left(\frac{s+1}{s^2+1} \right)$.

(2 × 15 = 30)

