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Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, SEPTEMBER 2024

First Semester

VECTOR ANALYSIS, DIFFERENTIAL EQUATIONS, FOURIER SERIES AND INTEGRAL TRANSFORMS

(Complementary Mathematics for B.Sc. Electronics/Computer Science)

[2013—2016 Admissions]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*

Each question carries 1 *mark.*

1. Give an example of a vector field.
2. Define the Laplace operator.
3. Define Curl of a vector field.
4. Test whether the equation $\left(x^2 - ay\right) dx = \left(ax - y^2\right) dy$ exact.
5. Write the form of the wave equation.
6. Write the formulas for the Fourier co-efficients for a periodic function f with period 2π.
7. Define even and odd functions with examples.
8. Write the Laplace transform of the function $f(t) = 1$, when $t \geq 0$.
9. Write the inverse Laplace transform of $\frac{5}{s+3}$.
10. State the shifting property of inverse Laplace Transform.

(10 × 1 = 10)

Turn over

Part B

Answer any **eight** *questions.*

Each question carries 2 *marks.*

11. Find ∇f, where $f = \frac{x}{y}$.

12. Find the curl of $v = yzi + 3zxj + zk$.

13. State Green's theorem in the plane.

14. Find the rectangular co-ordinates of the point with spherical co-ordinates $(\rho, \theta, \phi) = (4, \pi/3, \pi/4)$.

15. Solve the equation $\left(e^y + 1\right) \cos x dx + e^y \sin x \, dy = 0$.

16. Solve the equation $\sin x \frac{dy}{dx} - y \cos x + y^2 = 0$.

17. Solve the equation $\frac{d^2y}{dx^2} = \frac{3dy}{dx} + 2y = 0$.

18. Sketch the function $f(x) = x, -\pi < x < \pi$.

19. Write the Euler formulas for calculating the Fourier co-efficients of a period function with period 2L.

20. Write the Laplace transform of $\cos^2 t$.

21. Find the inverse transform of $\frac{12}{s^2 + 16}$.

22. Find the Laplace transform of $2te^t$.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*

Each question carries 4 *marks.*

23. Find the directional derivative of $f = x^2 + y^2 + z^2$ at $P : (2, -2, 1)$ the direction of $a = [-1, -1, 0]$.

24. Calculate $\int_C F \cdot dr$ where $F = [y^3, x^3]$, C the parabola $Y = 5x^2$ from $A(0, 0)$ to B (2, 20).

25. Describe the region of integration and evaluate $\int_0^1 \int_x^{2x} (x+y)^2 \, dydx$.

26. Solve the equation $(1-x^2)\frac{dy}{dx} - xy = x^2y^2$.

27. By finding the integrating factors solve the equation $(1+xy)\,ydx + (1-xy)\,xdy = 0$.

28. Find the Fourier Series of the following periodic function with period 2π.

$$f(x) = \begin{cases} 1 \text{ if } & -\pi/2 < x < \pi/2 \\ 0 \text{ if } & \pi/2 < x < 3\pi/2. \end{cases}$$

29. Find $F(t)$ if $L\{F(t)\} = \log_e \frac{s^2+1}{(s-1)^2}$.

30. Using Partial fractions find F (*t*) if $L\{F(t)\} = \frac{s+12}{s^2+4s}$.

31. Find the Laplace transform of $t^2 \sinh 2t$.

(6 × 4 = 24)

Turn over

Part D

Answer any **two** *questions.*

Each question carries 15 *marks.*

32. Verify stoke's theorem by evaluating the line integral and double integral for

$F(x, y, x) = x^2 i + y^2 j + z^2 k$, σ is the portion of the cone $z = \sqrt{x^2 + y^2}$ below the plane $z = 1$.

33. (a) Solve $\frac{d^2y}{dx^2} + 2y = x^2 e^{3x} + e^x \cos 2x$.

(b) Solve the system of equation :

$$\frac{dx}{dt} + 2y = -\sin t, \frac{dy}{dt} = 2x + \cos t.$$

34. (a) Solve the equation

$$yz\, p + zxq = xy.$$

(b) Obtain the complete solution of $pq + p + q = 0$.

35. Find the Fourier series for $|x|$ in $[-\pi, \pi]$ and deduce that $1 + \frac{1}{3^2} + \frac{1}{5^2} + ... = \pi^2/8$.

(2 × 15 = 30)