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Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, SEPTEMBER 2024

First Semester

Complementary Course—DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course for Physics/Chemistry/Petrochemicals/Geology/Food Science and Quality Control/Computer Maintenance and Electronics)

[2013—2016 Admissions]

Time : Three Hours

Maximum Marks : 80

Part A

Answer all questions.

Each question carries 1 mark.

1. Does $\lim_{x \rightarrow 0} \frac{x}{|x|}$ exists.
2. State Sandwich theorem for limits.
3. Find the slope of the tangent line to $f(x) = x + \frac{9}{x}$ at $x = -3$.
4. State the first derivative test for local extreme values.
5. Define the term critical points associated with a function f .
6. State mean value theorem.
7. Write the domain and range of $w = \sin(xy)$.
8. Define the partial derivative of $f(x, y)$ with respect to y at (x_0, y_0) .

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9. What is $\cos(ix)$ and $\sin(ix)$.
10. What is $\cosh(x + iy)$.

(10 × 1 = 10)

Part B

*Answer any **eight** questions.*

Each question carries 2 marks.

11. Find $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x+3}-2}$.

12. Find $\lim_{v \rightarrow 2} \frac{v^3-8}{v^4-16}$.

13. Evaluate $\lim_{x \rightarrow 0^+} \frac{x^2-3x+2}{x^3-2x^2}$ as :

(a) $x \rightarrow 0^+$.

(b) $x \rightarrow 2^+$.

14. Find the derivative of :

$$u = \frac{5x+1}{2\sqrt{x}}.$$

15. Find the absolute maxima and minima of $f(x) = 4 - x^2, -3 \leq x \leq 1$.

16. Check whether the function $f(x) = \frac{x^3}{3} - 3x$ satisfies the hypothesis of Rolle's theorem on $[-3, 3]$.





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17. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x, y) = x^2 - xy + y^2$.
18. Find f_{xy} and f_{yx} for $f(x, y) = xe^y + y + 1$.
19. Find $\frac{dy}{dx}$ if $x^2 + \sin y - 2y = 0$.
20. Prove that $\cosh^2 x - \sinh^2 x = 1$.
21. Separate $\cos(\alpha + i\beta)$ into real and imaginary parts.
22. Show that $\log(-1) = i\pi$.

(8 × 2 = 16)

Part C

Answer any **six** questions.

Each question carries 4 marks.

23. (a) Find $\lim_{x \rightarrow 3^-} \frac{1}{|x - 3|}$.
- (b) Let $f(x) = \frac{x^3 - 1}{x - 1}$, find $\lim_{x \rightarrow 1} f(x)$ and sketch the graph of $f(x)$.
24. (a) Find $\lim_{x \rightarrow -\infty} \frac{x + 4x^3}{1 - x^2 + 7x^3}$.
- (b) Show that $f(x) = |x|$ is not differentiable at $x = 0$.
25. (a) Find the derivative of $y = \frac{1 + \operatorname{cosec}(x^2)}{1 - \cos(x^2)}$.
- (b) Use implicit differentiation to find $\frac{dy}{dx}$ where $x^3 + y^3 = 3xy$.

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26. Find all absolute extreme of the function $f(x) = x^3 - 3x^2 + 4$ on the interval :

(a) $(-\infty, \infty)$.

(b) $(0, \infty)$.

27. Verify that the hypothesis of mean value theorem are satisfied by $f(x) = x^3 + x - 4$ on $[-1, 2]$.

28. (a) Let $z = e^{2x} \sin y$ find $\frac{\partial z}{\partial x}|(x, 0)$ and $\frac{\partial z}{\partial y}|(0, y)$.

(b) Let $z = (y^2 - 4x)$, find the rate of change of z with respect to x at $(2, 1)$ with y held fixed.

29. Show that $z = e^x \sin y + e^y \cos x$ satisfies Laplace's equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

30. If $\sin(\theta + i\phi) = \tan(x + iy)$, show that $\frac{\tan \theta}{\tanh \phi} = \frac{\sin 2x}{\sinh 2y}$.

31. Prove that $32 \cos^6 \theta = \cos 6\theta + 6 \cosh \theta + 15 \cos^2 \theta + 10$.

(6 × 4 = 24)

Part D

Answer any two questions.

Each question carries 15 marks.

32. (a) A spherical balloon is being inflated :

(i) Find the general formula for the instantaneous rate of change of the volume V with respect to the radius r , given that $V = \frac{4}{3} \pi r^3$.

(ii) Find the rate of change of V with respect to r at the instant when the radius is $r = 5$.





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- (b) Given that $y = \left(\frac{3x+2}{x}\right)(x^{-5} + 1)$ find $\frac{dy}{dx}$ at $x = 1$.
- (c) Find the slopes of the curves $y^2 - x + 1 = 0$ at the points $(2, -1)$ and $(2, 1)$.
33. (a) Find $\frac{dy}{dx}$, given that $x = \frac{a}{2}\left(t + \frac{1}{t}\right)$ $y = \frac{a}{2}\left(t - \frac{1}{t}\right)$.
- (b) If $x^2(1+y) = y^2(1+x)$ and $x \neq y$. Prove that $\frac{dy}{dx} + \frac{1}{(1+x)^2} = 0$.
- (c) Find the critical points of $f(x) = x^{1/3}(x-4)$. Identify intervals on which f is increasing and decreasing. Find the functions local and absolute extreme values.
34. (a) Suppose that $z = \sqrt{xy+y}$, $x = \cos \theta$, $y = \sin \theta$ use chain rule to find $\frac{dz}{d\theta}$ when $\theta = \pi/2$.
- (b) At what rate is the volume of box changing if its length is 8 ft and increasing at 3 ft/s, its width is 6ft and increasing at 2ft/sec. and its height 4ft and increasing at $\left| \text{ft} \right| \text{sec}$.
- (c) Let $T = x^2y - xy^3 + 2$, $x = r \cos \theta$, $y = r \sin \theta$. Find $\frac{\partial T}{\partial \theta}$ and $\frac{\partial T}{\partial r}$.
35. (a) Sum to infinity the series :

$$1 + \cos \theta + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + \dots$$

- (b) Expand $\sin^7 \theta$ in a series of sines of multiples of θ .

$$(2 \times 15 = 30)$$

