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Reg. No.................................

Name....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2024

Fourth Semester

Core Course—VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS

(Common for B.Sc. Mathematics Model I, II and B.Sc. Computer Applications)

[2013–2016 Admissions]

Time : Three Hours　　　　Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Write the vector equation of a plane through a point P_0 and normal to $n = Ai + Bj + Ck$.
2. How will you define the torsion of a smooth curve in space ?
3. Find the gradient of the function $g(x, y) = \frac{x^2}{2} - \frac{y^2}{2}$ at $\left(\sqrt{2}, 1\right)$.
4. State fundamental theorem of line integrals.
5. Define an exact differential form.
6. State Stoke's theorem.
7. Write an equation whose roots are negatives of the roots of the equation $x^3 + 6x^2 - 8x + 9 = 0$.
8. What is a reciprocal equation ?
9. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, express $\Sigma\alpha^2$ in terms of co-efficients.
10. Write Newton Raphson formula for solving algebraic and transcendental equations.

(10 × 1 = 10 marks)

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Find the unit tangent vector to the curve $r(t) = (2\cos t)i + (2\sin t)j + \sqrt{5}tk, 0 \le t \le \pi$.
12. Find the curvature K of the plane curve $r(t) = ti + (\ln\cos t)j, -\pi/2 < t < \pi/2$.

Turn over

13. Find the torsion for the curve $r(t) = \left(e^t \cos t\right) i + \left(e^t \sin t)\right) j + 2k$.

14. If $f(x, y, z) = x^2 + y^2 - 2z^2 + z \ln x$, find ∇f at (1, 1, 1).

15. Evaluate $\int_C (x + y) ds$ where C is the straight line segment $x = t$, $y = (1 - t)$, $z = 0$ from (0, 1, 0) to (1, 0, 0).

16. Show that $F = (2x - 3)i - 2j + (\cos z)\, k$ is not conservative.

17. Solve the equation $x^3 + 4x^2 - 12x - 27 = 0$ given that its roots are in G.P.

18. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, prove that $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha) = r - pq$, where $\alpha + \beta + \gamma = -p, \alpha\beta + \beta\gamma + \gamma\alpha = q, \alpha\beta\gamma = -r$.

19. Find the equation whose roots are 2 less than the roots of the equation $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$.

20. If the roots of the equation $x^3 + px^2 + qx + r = 0$ are in A.P. Show that $2p^3 - 9pq + 27r = 0$.

21. State Descarte's rule of signs.

22. Explain bisection method for solving algebraic and transcendental equations.

(8 × 2 = 16 marks)

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Find the binormal vector for the space curve $r(t) = \left(e^t \cos t\right) i + \left(e^t \sin t\right) j + 2k$.

24. Write the acceleration a in the form $a_T T + a_N N$ without finding T and N at t = 1, for the curve $r(t) = (t + 1)i + 2tj + t^2 k$.

25. Find the point on the curve $r(t) = (5 \sin t)i + (5 \cos t) j + 12tk$ at a distance of 26π units along the curve from the origin in the direction of increasing arc length.

26. Find the work done by $F = \left(y - x^2\right) i + \left(z - y^2\right) j + \left(x - z^2\right) k$ over the curve $r(t) = ti + t^2 j + t^3 k, 0 \le t \le 1$ from (0, 0, 0) to (1, 1, 1).

27. Find a potential function for the field $F = (y \sin z)i + (x \sin z)j + (xy \cos z)k$.

28. Use Green's theorem to find the counterclockwise circulation and outward flux for the field $F = (x - y)\, i + (y - x)j$ over the curve C : the square bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$.

29. Solve the equation $x^4 + 20x^3 + 143x^2 + 430x + 462 = 0$ by removing its second term.

30. Solve the equation $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

31. Use bisection method to find a root of $x^3 - x - 11 = 0$ correct to two decimal places which lies between 2 and 3.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at (1, 1, 0) in the direction of

    $A = 2i - 3j + 6k$.

    (b) In what direction does f changes most rapidly at (1, 1, 0) and what are the rates of change in these directions.

    (c) Find the plane tangent to the surface $z = x \cos y - ye^x$ at (0, 0, 0).

33. (a) Find the flux of $F = yzj + z^2k$ outward through the surface S cut from the cylinder $y^2 + z^2 = 1$, $z \geq 0$ by the planes $x = 0$ and $x = 1$.

    (b) Use Stoke's theorem to calculate the circulation of the field $F = yi + xzj + x^2k$ around the curve C which is the boundary of the triangle cut from the plane $x + y + z = 1$ by the first octant, counterclockwise when viewed from above.

34. (a) Solve the Cardan's method $x^3 - 18x - 35 = 0$.

    (b) Solve the Ferrari's method $x^4 + 6x^3 + 14x^2 + 22x + 5 = 0$.

35. (a) Use Newton Raphson method to find a root of $3x = 1 + \cos x$ correct to three decimal places.

    (b) Use iteration method to find a root of the equation $x^4 - x - 13 = 0$ correct to three significant figures.

(2 × 15 = 30)