FOURTH SEMESTER M.Sc. MATHEMATICS
ME800401-DIFFERENTIAL GEOMETRY
MULTIPLE CHOICE QUESTIONS

1. The level set of the function $f(x_1, x_2) = {x_1}^2 + \frac{{x_2}^2}{4}$ at height 1 is
   A. a circle
   B. a pair of straight lines
   C. a parabola
   D. an ellipse
2. The graph of the function $f(x_1) = {x_1}^2$ is
   A. a circle
   B. a parabola
   C. an ellipse
   D. a paraboloid
3. Which of the following is false about a function : $\mathbb{R}^{n+1} \to \mathbb{R}$?
   A. The level set of f at height $c \in \mathbb{R}$ is the solution set of the equation $f(x_1, x_2, \dots, x_{n+1}) = c$.
   B. The graph of f is a subset of $\mathbb{R}^{n+2}$
   C. The level set of f at height $c \in \mathbb{R}$ is always non-empty
   D. The graph of f is a level set for some function $F: \mathbb{R}^{n+2} \to \mathbb{R}$
4. The dimension of the vector space ${\mathbb{R}_p}^{n+1}$is
   A. n+2
   B. n+1
   C. p(n+1)
   D. n
5. If $\mathbf{v} = (p, v)$ and $\mathbf{w} = (p, w)$ are two vectors in ${\mathbb{R}_p}^{n+1}$, which of the following is not true?
   A. $\mathbf{v} + \mathbf{w} = (p, v + w)$
   B. $\mathbf{v} \cdot \mathbf{w} = (p, v \cdot w)$
   C. c$\mathbf{v} = (p, cv)$ where c is a scalar
   D. $\mathbf{v} \times \mathbf{w} = (p, v \times w)$

6. If $\mathbf{v} = (p, v)$ and $\mathbf{w} = (q, w)$ are two vectors in $\mathbb{R}^{n+1}$at points p and q respectively, then which of the following is true?
   A. $\mathbf{v} + \mathbf{w} = (p, v + w)$
   B. $\mathbf{v} + \mathbf{w} = (q, v + w)$
   C. $\mathbf{v} + \mathbf{w} = (p + q, v + w)$
   D. Not defined
7. The length of the vector $\mathbf{v} = (p, v)$ where p is a point in $\mathbb{R}^4$ and $v = (-1, 2, 4, -2)$ is
   A. 3
   B. 9
   C. 25
   D. 5
8. The angle between the vectors $\mathbf{v} = (p, 1, 0)$ and $\mathbf{w} = (p, 1, 1)$ at a point p in $\mathbb{R}^2$ is
   A. 0
   B. $\frac{\pi}{2}$
   C. $\frac{\pi}{4}$
   D. $\frac{\pi}{6}$
9. Which of the following is not true about a parametrized curve $\alpha: I \to \mathbb{R}^{n+1}$?
   A. α is smooth
   B. I is an open interval in $\mathbb{R}$
   C. The velocity vector at time $t \in I$ of α is tangent to α at $\alpha(t)$
   D. α is always an integral curve
10. Let $\mathbf{X}$ be a smooth vector field defined on an open set $U \subseteq \mathbb{R}^{n+1}$and let $p \in U$. If α is the maximal integral curve of $\mathbf{X}$ through p and β is any integral curve of $\mathbf{X}$ through p, then
   A. $\alpha = \beta$
   B. α is the restriction of β
   C. β is the restriction of α
   D. Domains of α and β are the same

11. The gradient of the function $f(x_1, x_2) = x_1 + x_2$ at (1, 0) is
   A. (1, 0, 1, 1)
   B. (1, 0, 1, 0)
   C. (1, 0, 0, 1)
   D. (1, 0, 0, 0)
12. The divergence of the vector field $\mathbf{X}(x_1, x_2) = (x_1, x_2, -x_2, x_1)$ is
   A. 0
   B. 1
   C. 2
   D. 4
13. Which of the following is not true?
   A. Associated with each smooth function $f: U \to \mathbb{R}$ where U open in $\mathbb{R}^{n+1}$ there is always a smooth vector field on U
   B. The gradient of a smooth function $f: U \to \mathbb{R}$ (U open in $\mathbb{R}^{n+1}$) at $p \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at p.
   C. The set of all vectors tangent to $f^{-1}(c)$ at a point $p \in U$ is always a proper subset of $[\nabla f(p)]^{\perp}$
   D. At each regular point p on a level set $f^{-1}(c)$ of a smooth function f there is a well-defined tangent space
14. Let $f: U \to \mathbb{R}$ (U open in $\mathbb{R}^{n+1}$) be a smooth function. A point $p \in \mathbb{R}^{n+1}$ such that $\nabla f(p) \neq 0$ is called
   A. a critical point of f
   B. a regular point of f
   C. a point of inflection
   D. saddle point of f
15. If $f: U \to \mathbb{R}$ (U open in $\mathbb{R}^{n+1}$) is a smooth function and $\alpha: I \to \mathbb{R}^{n+1}$ is a parametrized curve, then
   A. $\frac{d}{dt}(f \circ \alpha)(t) = \nabla f\big(\alpha(t)\big) \cdot \dot{\alpha}(t), \forall t \in I$
   B. $\frac{d}{dt}(f \circ \alpha)(t) = \|\nabla f(\alpha(t))\|^2, \forall t \in I$
   C. Both (A) and (B) are true
   D. None of the above

16. Which of the following is not true?

A. A parametrized curve can cross itself

B. An integral curve of a vector field can cross itself

C. At each point of a parametrized curve there is a vector tangent to the curve

D. An integral curve is always a parametrized curve

17. A 1-surface in $\mathbb{R}^2$ is called

A. a plane curve

B. simply a surface

C. a line

D. a plane

18. Let S be an oriented 3-surface in $\mathbb{R}^4$with orientation **N** and let $p \in S$. An ordered orthonormal basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ for the tangent space to S at p is said to be left-handed if the determinant

$det\begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ N(p) \end{pmatrix}$ is

A. non-zero

B. equal to zero

C. positive

D. negative

19. The surface of revolution obtained by rotating the curve $x_2 = 1$ about the x_1 axis is

A. a plane

B. a cylinder

C. a sphere

D. a parabola

20. A 1-plane in $\mathbb{R}^2$ is usually called

A. a plane curve

B. simply a surface

C. a plane

D. a line

21. The maximum and minimum values of the function $g(x_1, x_2) = {x_1}^2 + 2x_1x_2 + {x_2}^2$ on the circle ${x_1}^2 + {x_2}^2 = 1$ is

A. 2, 0

B. $\frac{1}{2}$, 0

C. 2, 1

D. 1, 0

22. The set S of all unit vectors at all points of $\mathbb{R}^2$ forms

A. a 1-surface in $\mathbb{R}^2$

B. a 2-surface in $\mathbb{R}^3$

C. a 3-surface in $\mathbb{R}^4$

D. a 4-surface in $\mathbb{R}^5$

23. The dimension of the tangent space at a point on an n-surface in $\mathbb{R}^{n+1}$ is

A. n+1

B. n

C. n-1

D. 1

24. Number of orientations on a connected n-surface in $\mathbb{R}^{n+1}$is

A. 0

B. 1

C. 2

D. n

25. Let S be an oriented n-surface in $\mathbb{R}^{n+1}$ with the orientation **N** and let $p \in S$. An ordered basis $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ for the tangent space to S at p is said to be consistent with the orientation **N** if the determinant $det\begin{pmatrix} v_1 \\ \vdots \\ v_n \\ N(p) \end{pmatrix}$ is

A. non-zero

B. equal to zero

C. positive

D. negative

26. Which of the following statement is correct?

(A) ${x_1}^2 + {x_2}^2 = 1$ is a circle in $\mathbb{R}^2$

(B) ${x_1}^2 + {x_2}^2 = 1$ is a cylinder in $\mathbb{R}^3$

(C) Both A and B

(D) None of the above

27. The domain of a parameterized curve is
 (A) $\mathbb{R}^n$
 (B) $\mathbb{R}^{n+1}$
 (C) An interval in $\mathbb{R}$
 (D) A subset of $\mathbb{R}^{n+1}$
28. Geodesic of an n-surafce S is a parametrized curve which is always
 (A) Parallel to S
 (B) Orthogonal to S
 (C) Parallel to $\mathbb{R}^{n+1}$
 (D) Orthogonal to $\mathbb{R}^{n+1}$
29. Let $\alpha: I \to \mathbb{R}^{n+1}$ is a parametrized curve with constant speed, then
 (A) $\alpha(t)$ and $\dot{\alpha}(t)$ are orthogonal
 (B) $\alpha(t)$ and $\dot{\alpha}(t)$ are parallel
 (C) $\dot{\alpha}(t)$ and $\ddot{\alpha}(t)$ are parallel
 (D) $\dot{\alpha}(t)$ and $\ddot{\alpha}(t)$ are orthogonal
30. The equation $-x_1^2 + x_2^2 + \cdots + x_{n+1}^2 = 0, x_1 > 0$ reprresents
 (A) a cone
 (B) a parabola
 (C) a hyperbola
 (D) a parabloid
31. Let S denote the cylinder ${x_1}^2 + {x_2}^2 = r^2$ of radius $r > 0$ in $\mathbb{R}^3$. Also α is a geodesic of S then α is of the form
 (A) $\alpha(t) = (r\cos(at+b), r\ \sin(at+b), ct+d)$
 (B) $\alpha(t) = (a + r\cos\ t\ , b +\ r\ \sin t, ct+d)$
 (C) $\dot{\alpha}(t) = (r\cos(at+b), r\sin(at+b), ct+d)$
 (D) None of the above
32. A parametrized curve α in a unit sphere $x_1^2 + x_2^2 + \cdots + x_{n+1}^2 = 1$ is given as $\alpha(t) = (\cos at)e_1 + (\sin at)e_2$, where $\{e_1, e_2\}$ is some orthogonal pair of unit vectors , $a \in \mathbb{R}$. Then α is
 (A) an $n+1$ sphere
 (B) a geodesic
 (C) maximal integral curve
 (D) none of the above
33. An n-surface S in $\mathbb{R}^{n+1}$ is said to be geodesically complete if every maximal geodesic in S has domain
 (A) $\mathbb{R}$
 (B) $\mathbb{R}^{n+1}$
 (C) an interval in $\mathbb{R}$
 (D) none of the above

34. A parametrized curve $\alpha: I \to S$ is a geodesic if
 (A) The velocity $\dot{\alpha}(t) = 0$ for all t
 (B) The acceleration $\ddot{\alpha}(t) = 0$ for all t
 (C) The covariant acceleration $(\dot{\alpha})' = 0$
 (D) None of the above
35. Which of the following are true bout a parallel transport $P_\alpha: S_p \to S_q$
 (A) P_α is a linear map.
 (B) P_α is one-one and onto

(C) $P_\alpha(v).P_\alpha(w) = v.w$ for all v, w in S_p
(D) All of the above

36. For each n-surface S, there exists a Gauss map, which is of the form
(A) $N: S \to S^n$
(B) $N: R \to \mathbb{R}^n$
(C) $N: S^n \to \mathbb{R}^n$
(D) None of the above

37. Let S be a compact, connected and oriented n-surface in $\mathbb{R}^{n+1}$ exhibited as a level set $f^{-1}(c)$ of a smooth function $f: R^{n+1} \to \mathbb{R}$ with $\nabla f(p) \neq 0$ for all $p \in S$, then
(A) the Gauss map is one-one
(B) the Gauss map is onto
(C) the Gauss map is a bijection
(D) none of the above

38. The image of Gauss map $N(S) = \{q \in S^n : q = N(p)\ for\ some\ p \in S\}$ is called ________ image of oriented surface
(A) parabolic image
(B) spherical image
(C) circular image
(D) none of the above

39. The equation $-x_1 + x_2^2 + \cdots + x_{n+1}^2 = 0$ represents a
(A) parabola
(B) parabloid
(C) sphere
(D) cone

40. The acceleration of the geodesic is
(A) keeping it in surface
(B) tangential to surface
(C) orthogonal to surface
(D) none of the above

41. If an n-surface S contains a straight-line segment $\alpha(t) = p + tv, t \in I$, then it is a
(A) maximal integral curve in $\mathbb{R}$
(B) Geodesic in S
(C) tangential vector filed in S
(D) none of the above

42. Let X and Y be smooth vector fields along the parametrized curve $\alpha: I \to \mathbb{R}^{n+1}$ and let $f: I \to \mathbb{R}$ be a smooth function along α then
(A) $(X \dot{+} Y) = \dot{X} + \dot{Y}$
(B) $\left(f'(X)\right) = f'X + f\dot{X}$
(C) $(X.Y)' = \dot{X}.Y + X.\dot{Y}$
(D) All of the above

43. What is the acceleration of the parametrization $\alpha(t) = (t, t^2)$
(A) $(\alpha(t); 0,2)$
(B) $(\dot{\alpha}(t); 0,2)$
(C) $(\alpha(t); 1,2)$
(D) $(\dot{\alpha}(t); 1,2)$

44. Find the velocity of $\alpha(t) = (\cos t, \sin t)$
(A) $(\alpha(t); -\sin t, \cos t)$

(B) $(\alpha(t); -\cos\ t, \sin t)$
(C) $(\dot{\alpha}(t); -\sin t, \cos t)$
(D) $(\dot{\alpha}(t); -\cos\ t, \sin t)$

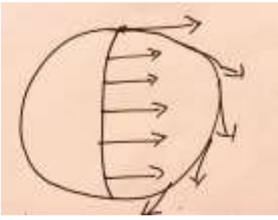
45. Find the speed of $\alpha(t) = (\cos 3t, \sin 3t)$
(A) -3
(B) 0
(C) 3
(D) None of the above

46. Consider the cylinder $x_2^2 + \cdots + x_{n+1}^2 = 1$. Define $f: \mathbb{R}^{n+1} \to \mathbb{R}$ by $f(x_1, \dots, x_{n+1}) = x_2^2 + \cdots + x_{n+1}^2$. Find $\|\nabla f(p)\|$
(A) 0
(B) 1
(C) 2
(D) 3

47. Which of the following is/are the property/properties of Levi-Civita parallelism
(A) If X and Y are parallel along α, then the angle $\cos^{-1}({X.Y}/{\|X\|.\|Y\|})$ is constant along α
(B) If X and Y are parallel along α,then so are $X + Y$ and cX for all $c \in \mathbb{R}$
(C) The velocity vector field along a parametrized curve α in S is parallel if and only if α is a geodesic
(D) All of the above

48. Let S be an n-surface in $\mathbb{R}^{n+1}$. Let $\alpha: I \to S$ be a parametrized curve and let X and Y be vector fields tangent to S along α. Then
(A) $(X + Y)' = X' + Y'$
(B) $(fX)' = f'X + fX'$
(C) both A and B
(D) none of the above

49. The following figure represents



(A) Levi-Civita parallel vector field
(B) Eucledian parallel vector field
(C) both (A) and (B)
(D) none of the above

50. Meridians and Parallels always meet
(A) Parallely
(B) Orthogonally
(C) Tangentially
(D) They never meet

51. The Weingarten map is also known as ……………
(A) Surface homomorphism
(B) Shape operator
(C) Tangent space homomorphism
(D) Euclidian homomorphism

52. The Weingarten map is given by for $\boldsymbol{v} = \dot{\alpha}(t_0)$. $L_p(\boldsymbol{v}) = \cdots$

(A) $(N \circ \alpha)'(t_0)$

(B) $-(\mathrm{N} \circ \alpha)'(t_0)$

(C) $(\mathrm{N} \circ^{\cdot} \alpha)(t_0)$

(D) $-(\mathrm{N} \circ^{\cdot} \alpha)(t_0)$

53. For every parametrized curve $\alpha: I \to S$ with $\alpha^{\cdot}(t_0) = \mathbf{v}$, $L_p(\mathbf{v}) \cdot \mathbf{v}$ is -----

(A) The normal component of acceleration

(B) The tangential component of acceleration

(C) the absolute value or acceleration

(D) None of these

54. Let S be a sphere of radius r, oriented by its outward normal and let α be a unit speed parametrized curveon S. Then the normal component of acceleration of α is given by

(A) 0
(B) 1
(C) $\frac{1}{r}$
(D) $-\frac{1}{r}$

55. Let $f: \mathbf{R}^2 \to \mathbf{R}$, given by $f(x, y) = 2x^2 + 3y^2$, $p = (1, 0)$, $\mathbf{v} = (p, 2, 1)$. Then $\nabla_{\mathbf{v}} f$ =……

(A) 14

(B) 10

(C) 8

(D) 6

56. Let $f: \mathbf{R}^2 \to \mathbf{R}$, given by $f(x, y) = x^2 - y^2$, $p = (1, 1)$, $\mathbf{v} = (p, \cos\theta, \sin\theta)$. Then $\nabla_{\mathbf{v}} f$ =……

(A) $2(\cos\theta - \sin\theta)$

(B) $2(\cos\theta + \sin\theta)$

(C) $\sin 2\theta$

(D) $\cos 2\theta$

57. Which of the following is true?

$(i)\ \nabla_v(\boldsymbol{X} + \boldsymbol{Y}) = \nabla_v \boldsymbol{X} + \nabla_v \boldsymbol{Y}$ $\quad (ii)\ \nabla_v(f\boldsymbol{X}) = (\nabla_v f)\boldsymbol{X}(p) + f(p)(\nabla_v \boldsymbol{X})$

(A) Only (i) is true
(B) Only (ii) is true
(C) Both are false
(D) Both are true

58. Curvature of straight line is

(A) 0
(B) 1
(C) -1
(D) None

59. If two curves have same curvature then, these curves are

(A) Equal to each other
(B) Perpendicular to each other
(C) Congruent
(D) None

60. The magnitude of curvature is

(A) T
(B) K
(C) C
(D) None

61. The curvature K is always

(A) Zero
(B) Positive
(C) Negative
(D) None

62. Let S be a plane curve, $p \in S$ such that the radius of curvature at p is non zero, and let C be the circle ofcurvature at p. Then which of the following statement is true?

$(i) S_p = C_p$ (ii) orientation normals of S and C at p are equal

(A) Only (i) is true

(B) Only (ii) is true

(C) Both are false

(D) Both are true

63. Let $(t) = ((t), (t))$ be a local parametrization of a plane curve C. Then the curvature of C at (t) isgiven by ………….

(A) $\frac{x'y''-x''y'}{(x'^2+y'^2)^{\frac{3}{2}}}$

(B) $\frac{x'y''+x''y'}{(x'^2-y'^2)^{\frac{3}{2}}}$

(C) $\frac{x'y''-x''y'}{(x'^2+y'^2)^{\frac{2}{3}}}$

(D) $\frac{x'y''+x''y'}{(x'^2-y'^2)^{\frac{2}{3}}}$

64. A necessary and sufficient condition for the existence of a global parametrization for a plane curve C is

(A) C is compact

(B) C is connected

(C) C is connected and compact

(D) None of these

65. A necessary condition for a unit speed parametrized curve α to be a global parametrization of aconnected plane curve C is

(A) α is injective

(B) α is periodic

(C) α is either injective or periodic

(D) α is both injective and periodic

66. Let ω be an exact 1-form. Then the integral of ω over a compact connected oriented plane curve is

(A) Always zero

(B) Always one

(C) $\omega(0)$

(D) Not defined

67. Let μ be the 1-form on $\mathbb{R}^2 - \{0\}$ defined by $\mu = -\frac{x_2}{{x_1}^2+{x_2}^2}\, dx_1 + \frac{x_1}{{x_1}^2+{x_2}^2}\, dx_2$ and let C denote the ellipse $\frac{{x_1}^2}{a^2} + \frac{{x_2}^2}{b^2} = 1$, oriented by its inward normal. Then the value of $\int_C \mu$ is

(A) 0

(B) 2π

(C) π

(D) -2π

68. Let η be the 1-form on $\mathbb{R}^2 - \{0\}$ defined by $\eta = -\frac{x_2}{{x_1}^2+{x_2}^2}\, dx_1 + \frac{x_1}{{x_1}^2+{x_2}^2}\, dx_2$. Then for $\alpha: [a, b] \to \mathbb{R}^2 - \{0\}$ any closed piecewise smooth parametrized curve in $\mathbb{R}^2 - \{0\}$, $\int_\alpha \eta$ is

(A) 0

(B) $2\pi k$, for some integer k

(C) π

(D) $-2\pi k$, for some integer k

69. A parametrized curve $\alpha: [a, b] \to \mathbb{R}^{n+1}$ is said to be closed if ……

(A) $\alpha(a) = \alpha(b)$
(B) $\alpha(a) \neq \alpha(b)$
(C) $\alpha(a) = \alpha(b) = 0$
(D) None

70. If $\beta: \tilde{I} \to \mathbb{R}^{n+1}$ be a reparameterization of α, then which of the following is true?

$$(i)\ I(\alpha) = I(\beta) \quad (ii) I(\alpha) \neq I(\beta)$$

(A) Only (i) is true

(B) Only (ii) is true

(C) Both are false

(D) Both are true

71. Find the length of the parametrized curve $\alpha : I \to \mathbb{R}^4$ given by $\alpha(t) = (cost, sint, cost, sint)$, where $I = [0,2\pi]$

(A) $2\sqrt{2}\pi$
(B) 0
(C) $-2\sqrt{2}\pi$
(D) 2π

72. Find the length of the parametrized curve $\alpha : I \to \mathbb{R}^3, \alpha(t) = (\cos 3t, \sin 3t, 4t), I = [-1, 1]$

(A) 5
(B) 0
(C) 10
(D) 20

73. Let f and g be smooth functions on the open set $U \subset \mathbb{R}^{n+1}$, then which of the following is true?

$(i)\ d(f+g) = df + dg\ (ii)\ d(fg) = gdf + fdg$

(A) Only (i) is true

(B) Only (ii) is true

(C) Both are false

(D) Both are true

74. Compute $\int_\alpha x_2 dx_1 - x_1 dx_2$, where $\alpha(t) = (2\cos t, 2\sin t), 0 \leq t \leq 2\pi$

(A) 2π
(B) 8π
(C) 4π
(D) π

75. Compute $\int_C -x_2 dx_1 + x_1 dx_2$, where C is the ellipse $\frac{{x_1}^2}{a^2} + \frac{{x_2}^2}{b^2} = 1$ oriented by its inward normal
(A) $2\pi ab$
(B) πab
(C) 0
(D) $4\pi ab$

76. For $n = 1$, $L_p(\mathbf{v})$ is …………

(A) $= \boldsymbol{\kappa}(p)\ \mathbf{v}$ (B) $\neq \boldsymbol{\kappa}(p)\ \mathbf{v}$ (C) $> \boldsymbol{\kappa}(p)\ \mathbf{v}$ (D) $< \boldsymbol{\kappa}(p)\ \mathbf{v}$

77. When $\|\mathbf{v}\| = 1, k(\mathbf{v}) = L_p(\mathbf{v}) \cdot \mathbf{v}$ is called …………

(A) the Gaussian curvature at p (B) the normal curvature at p

(C) the principal curvature at p (D) the mean curvature at p

78. The normal section $\mathcal{N}(\mathbf{v})$ is just a copy of

(A) $\mathbb{R}^{n+1}$ (B) $\mathbb{R}^n$ (C) $\mathbb{R}^2$ (D) $\mathbb{R}$

79. Eigen values of the Weingarten map L_p are called …………..

(A) Gaussian curvatures at p (B) principal curvatures at p

(C) normal curvatures at p (D) mean curvatures at p

80. Let S denote the hyperboloid $-x_1^2 + x_2^2 + x_3^2 = 1$ in $\mathbb{R}^3$, oriented by its outward normal. Let $p = (0,0,1)$. Then the principal curvatures of S at p are

(A) $-1, 0, 1$ (B) $-1,0$ (C) $0,1$ (D) $-1,1$

81. If $\{k_1(p), k_2(p), \dots, k_n(p)\}$ are principal curvatures with corresponding orthogonal principal curvature directions $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ at a point p of an oriented n-surface S in $\mathbb{R}^{n+1}$, then the normal curvature $k(\mathbf{v})$ for $\mathbf{v} \in S_p$ is given by

(A) $\sum_{i=1}^n k_i(p)(\mathbf{v} \cdot \mathbf{v}_i)$ (B) $\sum_{i=1}^n k_i(p)(\mathbf{v} \cdot \mathbf{v}_i)^2$

(C) $\sum_{i=1}^n k_i(p)(\mathbf{v} \cdot \mathbf{v}_i)^n$ (D) $\sum_{i=1}^n k_i(p)(\mathbf{v} \cdot \mathbf{v}_i)^{n+1}$

82. Let $\{k_1(p), k_2(p), \dots, k_n(p)\}$ are principal curvatures with corresponding orthogonal principal curvature directions $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ at a point p of an oriented n-surface S in $\mathbb{R}^{n+1}$. For $\mathbf{v} \in \mathrm{S}_p$ and for $i = 1,2, \dots, n$, let θ_i be the angle between $\mathbf{v}$ and $\mathbf{v}_i$. Then the normal curvature $k(\mathbf{v})$ is given by

(A) $\sum_{i=1}^{n} k_i(p) \cos\theta_i$ (B) $\sum_{i=1}^{n} k_i(p) \sin\theta_i$

(C) $\sum_{i=1}^{n} k_i(p) \cos^2\theta_i$ (D) $\sum_{i=1}^{n} k_i(p) \sin^2\theta_i$

83. The quadratic form associated with a self adjoint linear transformation L is defined by

(A)$L(v) \cdot v$ (B) $[L(v) \cdot v]^2$ (C) $L(v) + v$ (D) $L(v) - v$

84. First fundamental form is always

(A) indefinite (B) semi-definite

(C) positive definite (D) negative definite

85. Which formula is used to recover the self adjoint linear transformation L from the quadratic form Q

(A) $L(v) \cdot w = \frac{1}{2}[Q(v+w) + Q(v) + Q(w)]$ (B) $L(v) \cdot w = \frac{1}{2}[Q(v+w) - Q(v) - Q(w)]$

(C) $L(v) \cdot w = \frac{1}{2}[Q(v+w) + Q(v) - Q(w)]$ (D) $L(v) \cdot w = \frac{1}{2}[Q(v+w) - Q(v) + Q(w)]$

86. When the quadratic form Q is such that $Q(v) \geq 0 \;\; \forall\, v$, it is said to be

(A) positive semi-definite (B) indefinite

(C) positive definite (D) negative definite

87. When the quadratic form Q is such that $Q(v) \leq 0 \;\; \forall\, v$, it is said to be

(A) positive definite (B) indefinite

(C) negative semi-definite (D) negative definite

88. When all the principal curvatures at p are positive, then the second fundamental form at p is

(A) positive semi-definite (B) negative semi-definite

(C) positive definite (D) negative definite

89. When all the principal curvatures at p are negative, then the second fundamental form at p is

(A) positive definite (B) negative definite

(C) positive semi-definite (D) negative semi-definite

90. Let $S = f^{-1}(c)$ be an n-surface in $\mathbb{R}^{n+1}$, oriented by $\nabla f / \|\nabla f\|$. For $\mathbf{v} = (p, v_1, v_2, \ldots, v_{n+1})$, a vector tangent to S at $p \in S$, the second fundamental form of S at p on $\mathbf{v}$ is given by

(A) $\frac{1}{\|\nabla f(p)\|^2} \sum_{i,j=1}^{n+1} \frac{\partial^2 f}{\partial x_i \partial x_j}(p) v_i v_j$ (B) $\frac{1}{\|\nabla f(p)\|} \sum_{i,j=1}^{n+1} \frac{\partial^2 f}{\partial x_i \partial x_j}(p) v_i v_j$

(C) $\frac{-1}{\|\nabla f(p)\|} \sum_{i,j=1}^{n+1} \frac{\partial^2 f}{\partial x_i \partial x_j}(p) v_i v_j$ (D) $\frac{1}{\|\nabla f(p)\|^2} \sum_{i,j=1}^{n+1} \frac{\partial^2 f}{\partial x_i \partial x_j}(p) v_i v_j$

91. The determinant of the Weingarten map L_p is called …………..

(A) Gauss-Kronecker curvature at p (B) principal curvature at p

(C) normal curvature at p (D) mean curvature at p

92. The trace of the Weingarten map L_p is called …………..

(A) normal curvature at p (B) principal curvature at p

(C) Gauss-Kronecker curvature at p (D) mean curvature at p

93. Let S be an oriented $n-$ surface in $\mathbb{R}^{n+1}$ and let $p \in S$. Let $\mathbf{Z}$ be any non-zero normal vector field on S such that $\mathbf{N} = \mathbf{Z}/\|\mathbf{Z}\|$ and let $\{\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}\}$ be any basis for S_p. Then the Gauss-Kronecker curvature at p is given by

(A) $\mathbf{K}(p) = (-1)^n \det\begin{pmatrix} \nabla_{\mathbf{v_1}}\mathbf{Z} \\ \vdots \\ \nabla_{\mathbf{v_n}}\mathbf{Z} \\ \mathbf{Z}(p) \end{pmatrix} \Big/ \|\mathbf{Z}(p)\|^2 \det\begin{pmatrix} \mathbf{v_1} \\ \vdots \\ \mathbf{v_n} \\ \mathbf{Z}(p) \end{pmatrix}$

(B) $\mathbf{K}(p) = (-1)^n \det\begin{pmatrix} \nabla_{\mathbf{v_1}}\mathbf{Z} \\ \vdots \\ \nabla_{\mathbf{v_n}}\mathbf{Z} \\ \mathbf{Z}(p) \end{pmatrix} \Big/ \|\mathbf{Z}(p)\|^n \det\begin{pmatrix} \mathbf{v_1} \\ \vdots \\ \mathbf{v_n} \\ \mathbf{Z}(p) \end{pmatrix}$

(C) $\mathbf{K}(p) = (-1)^n \det\begin{pmatrix} \nabla_{\mathbf{v_1}}\mathbf{Z} \\ \vdots \\ \nabla_{\mathbf{v_n}}\mathbf{Z} \\ \mathbf{Z}(p) \end{pmatrix} \Big/ \|\mathbf{Z}(p)\| \det\begin{pmatrix} \mathbf{v_1} \\ \vdots \\ \mathbf{v_n} \\ \mathbf{Z}(p) \end{pmatrix}$

(D) $\mathbf{K}(p) = (-1)^n \det\begin{pmatrix} \nabla_{\mathbf{v_1}}\mathbf{Z} \\ \vdots \\ \nabla_{\mathbf{v_n}}\mathbf{Z} \\ \mathbf{Z}(p) \end{pmatrix} \Big/ \|\mathbf{Z}(p)\|^{n+1} \det\begin{pmatrix} \mathbf{v_1} \\ \vdots \\ \mathbf{v_n} \\ \mathbf{Z}(p) \end{pmatrix}$

94. Let S be the ellipsoid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$ oriented by its outward normal. Then the Gaussian curvature of S is given by

(A) $\frac{1}{a^2b^2c^2\left(\frac{x_1^2}{a^4}+\frac{x_2^2}{b^4}+\frac{x_3^2}{c^4}\right)^2}$

(B) $\frac{1}{a^2b^2c^2\left(\frac{x_1^2}{a^2}+\frac{x_2^2}{b^2}+\frac{x_3^2}{c^2}\right)^2}$

(C) $\frac{1}{a^2b^2c^2\left(\frac{x_1^2}{a^2}+\frac{x_2^2}{b^2}+\frac{x_3^2}{c^2}\right)^{3/2}}$

(D) $\frac{1}{a^2b^2c^2\left(\frac{x_1^2}{a^4}+\frac{x_2^2}{b^4}+\frac{x_3^2}{c^4}\right)^{3/2}}$

95. If S and S' denote the same n-surface in $\mathbb{R}^{n+1}$ but with opposite orientations and if K and K' are the Gauss-Kronecker curvatures of S and S' respectively, then ……….

(A) $K' = (-1)^{n+1}K$ (B) $K' = (-1)^n K$ (C) $K' = (-1)^{n-1}K$ (D) $K' = (-1)^{n+2}K$

96. Let S be a compact connected oriented n-surface in $\mathbb{R}^{n+1}$. Which of the following is a necessary and sufficient condition for the Gauss-Kronecker curvature to be non zero for all $p \in S$.

(A) The second fundamental form is positive definite at each point.

(B) The first fundamental form is positive definite at each point.

(C) The second fundamental form is negative definite at each point.

(D) The second fundamental form is definite at each point.

97. Which of the following is not true for a parametrized n-surface in $\mathbb{R}^{n+k}$?

(A) It is a smooth map

(B) Its domain is an open set in $\mathbb{R}^n$

(C) Its domain is a disconnected set in $\mathbb{R}^n$

(D) It is regular

98. Let $\varphi: U \to \mathbb{R}^3$ be given by $\varphi(\theta, \phi) = (r\cos\theta\sin\phi, r\sin\theta\sin\phi, r\cos\phi)$ where $U = \{(\theta, \phi) \in \mathbb{R}^2 : 0 < \phi < \pi\}$ and $r > 0$. Then which of the following is true?

(A) φ is a parametrized 2-surface

(B) Image of φ is the 2-sphere of radius r in $\mathbb{R}^3$ with the north and south poles missing

(C) φ is not one to one

(D) All the above are true

99. Let $\alpha: I \to \mathbb{R}^2$ given by $\alpha(t) = (x(t), y(t))$ be a regular parametrized curve in $\mathbb{R}^2$ whose image lies above the x_1-axis. Then the parametrized surface of revolution obtained by rotating α about the x_1-axis is given by ….

(A) $\varphi: I \times \mathbb{R} \to \mathbb{R}^3, \varphi(t, \theta) = (x(t), y(t)\sin\theta, y(t)\cos\theta)$

(B) $\varphi: I \times \mathbb{R} \to \mathbb{R}^3, \varphi(t, \theta) = (x(t), y(t)\cos\theta, y(t)\sin\theta)$

(C) $\varphi: I \to \mathbb{R}^3, \varphi(t) = (x(t), y(t)\sin t, y(t)\cos t)$

(D) $\varphi: I \to \mathbb{R}^3, \varphi(t) = (x(t), y(t)\cos t, y(t)\sin t)$

100. Let $\varphi: U \to \mathbb{R}^4$ be given by $\varphi(\phi, \theta, \psi) = (\sin\phi\sin\theta\sin\psi, \cos\phi\sin\theta\sin\psi, \cos\theta\sin\psi, \cos\psi)$ where $U = \{(\phi, \theta, \psi) : \phi \in \mathbb{R}, 0 < \theta < \pi, 0 < \psi < \pi\}$. Then which of the following is true?

(A) φ is smooth and U is open

(B) φ is a parametrized 3-surface in $\mathbb{R}^4$

(C) Image of φ is contained in the unit 3-sphere in $\mathbb{R}^4$

(D) All the above are true

FOURTH SEMESTER M.Sc. MATHEMATICS
ME800401-DIFFERENTIAL GEOMETRY
MULTIPLE CHOICE QUESTIONS-ANSWER KEY

Module 1			Module 2			Module 3			Module 4	
Qn.No	Correct Choice		Qn.No	Correct Choice		Qn.No	Correct Choice		Qn.No	Correct Choice
1	D		26	C		51	B		76	A
2	B		27	C		52	D		77	B
3	C		28	B		53	A		78	C
4	B		29	D		54	D		79	B
5	B		30	A		55	C		80	D
6	D		31	A		56	A		81	B
7	D		32	B		57	D		82	C
8	C		33	A		58	A		83	A
9	D		34	C		59	C		84	C
10	C		35	D		60	B		85	B
11	A		36	A		61	B		86	A
12	A		37	B		62	D		87	C
13	C		38	B		63	A		88	C
14	B		39	B		64	B		89	B
15	A		40	A		65	C		90	C
16	B		41	B		66	A		91	A
17	A		42	D		67	B		92	D
18	D		43	A		68	B		93	B
19	B		44	A		69	A		94	A
20	D		45	C		70	A		95	B
21	A		46	C		71	A		96	D
22	C		47	D		72	C		97	C
23	B		48	C		73	D		98	D
24	C		49	A		74	B		99	B
25	C		50	B		75	A		100	D