IV SEMESTER MSc MATHEMATICS
Analytic Number Theory   MCQ

1. Euler totient function, $\varphi(n), n \geq 1$
   (a) takes only values 0,1 and -1.
   (b) takes value 1 for prime numbers.
   (c) is bounded.
   (d) takes value $p-1$ for prime numbers.

2. Which of the following expression for Euler totient function, $\varphi(n)$ is not true.
   (a) $\varphi(n) = \prod_{p/n} \quad (1 - \frac{1}{p})$
   (b) $\varphi(n) = \sum_{d/n} \quad \mu(d)\frac{n}{d}$
   (c) $\varphi(n) = \sum_{k=1}^{n} \quad [\frac{1}{(n,k)}]$
   (d) $\varphi(n) = \sum_{d/n} \quad \sum_{q=1}^{\frac{n}{d}} \quad \mu(d)$

3. Which of the following arithmetic functions is not a multiplicative function.
   (a) Power function, $f_\alpha(n) = n^\alpha$
   (b) Identity function, $I(n) = [\frac{1}{n}]$
   (c) Dirishlet product $f * g$ of two multiplicative functions.
   (d) Mangoldt function, $\Lambda(n)$

4. For a multiplicative function $f, f(1)$ equals,
   (a) 1
   (b) 0
   (c) -1
   (d) can be any value

5. Find the number of integers less than or equal to 100 which are relatively prime to 100.
   (a) 20
   (b) 40
   (c) 40
   (d) 60

6. Which arithmetic function is the dirichlet inverse of the Mobius function, $\mu(n)$
   (a) Euler totient function, $\varphi(n)$
   (b) $u(n) = 1$, for all n
   (c) Identity function, $I(n) = \left[\frac{1}{n}\right]$, for all n
   (d) $N(n) = n$, for all n

7. For the Mangoldt function $\Lambda(10)$ equals
   (a) $log(10)$
   (b) 1
   (c) 0

(d) $log(2)$

8. Average order of divisor function, $d(n)$ is
   (a) $log(n)$
   (b) n
   (c) $\frac{n^2}{2}$
   (d) $\sqrt{n}$

9. Value of $\zeta(2)$, where ζ is Riemann zeta function
   (a) 0
   (b) π
   (c) $\frac{\pi^2}{6}$
   (d) $\frac{6}{\pi^2}$

10. Average order of divisor function, $\sigma_\alpha(n)$, where $\alpha = 1$
   (a) $\frac{\pi^2 n}{12}$
   (b) $\frac{\pi^2 n}{6}$
   (c) $nlog(n)$
   (d) $log(n)$

11. Limit of density of lattice points visible from the origin is
   (a) $\lim_{r\to\infty} log\, r$, where r is the half of the length of the side of the square in plane with centre at origin.
   (b) $\frac{6}{\pi^2}$
   (c) $\frac{24}{\pi^2}r^2 + O(rlog(r))$, where r is as in option (a)
   (d) π

12. Probability that two integers a and b chosen random are relatively prime is
   (a) $gcd(a, b)$
   (b) $\frac{ab}{gcd(a,b)}$
   (c) π
   (d) $\frac{6}{\pi^2}$

13. Average order of $\mu(n)$ is
   (a) 1
   (b) 0
   (c) $log\, n$
   (d) $nlog\, n$

14. Average order of $\Lambda(n)$ is
   (a) 1
   (b) 0

(c) $log\ n$
(d) $nlog\ n$

15. Weight average of $\mu(n)$ is
(a) 1
(b) 0
(c) $log\ n$
(d) $log\ [n]!$

16. Weight average of $\Lambda(n)$ is
(a) 1
(b) 0
(c) $log\ n$
(d) $log\ [n]!$

17. Average order of $\varphi(n)$ is,
(a) $\frac{\pi^2}{6}$
(b) $log\ n$
(c) $\frac{3n}{\pi^2}$
(d) $nlog\ n$

18. Let F denotes a real/complex valued function defined on positive real axis$(0,+\infty)$: $F(x) = 0$ for $0 < x < 1$ and α, β be arithmetic functions. Let $\alpha \circ F$ be generalised convolution of α & F and $*$ be the dirichlet product. Then which of the following is true.
(a) $\alpha \circ \beta = \beta \circ \alpha$
(b) $\alpha \circ (\beta \circ F) = (\alpha * \beta) \circ F$
(c) $\alpha \circ (\beta \circ F) = (\alpha \circ \beta) \circ F$
(d) $\alpha \circ F = \alpha * F$

19. Which of the following is not true
(a) $log\ [x]! = \sum_{n\leq x} \quad \Lambda(n)[\frac{x}{n}]$
(b) $log\ [x]! = \sum_{p\leq x} \quad \alpha(p) log\ p, \alpha(p) = \sum_{m=1}^{\infty} \quad [\frac{x}{p^m}]$
(c) $log\ [x]! = log\ x + O(log\ x)$
(d) $log\ [x]! = xlog\ x - x + O(log\ x)$

20. For $x \geq 1, s > 1, O(x^{-s})$ is
(a) $O(x^{s-1})$
(b) $O(x^{s+1})$
(c) $O(x^{1-s})$
(d) $O(x^{2s})$

21. If $x \geq 1, \alpha > 0, O(x^{\alpha}) + O(x) = O(x^{\beta})$, then
(a) $\beta = max\{1, \alpha\}$
(b) $\beta = min\{1, \alpha\}$

(c) $\beta = 1$
(d) $\beta = \alpha$

22. Which of the following sum represents the number of lattice points in a $qd-$ plane that lie on the hyperbola, $qd = n, n = 1,2, ..., [x]$
    (a) $\sum_{n\leq x} \quad \mu(n)$, where μ is the Mobius function
    (b) $\sum_{n\leq x} \quad \Lambda(n)$. where Λ is the Mangoldt function
    (c) $\sum_{n\leq x} \quad \varphi(n)$, where φ is the Euler totient function
    (d) $\sum_{n\leq x} \quad d(n)$, where d is the divisor function.

23. Let f be a completely multiplicative function. then which of the following is the Dirichlet inverse of f
    (a) $f^{-1}(n) = \varphi(n)f(n)$, where φ is the Euler totient function
    (b) $f^{-1}(n) = \mu(n)f(n)$, where μ is the Mobius function
    (c) $f^{-1}(n) = \Lambda(n)f(n)$, where Λ is the Mangoldt function
    (d) $f^{-1}(n) = u(n)f(n)$, where u is the unit function

24. What is the identity element in the abelian group of all arithmetical functions f with $f(1) \neq 0$ under the operation Dirichlet product
    (a) f(n)=n, for all n
    (b) f(n)=1, for all n
    (c) f(n)=$\left[\frac{1}{n}\right]$, for all n, where $[x]$, is the greater inter less than or equal to x.
    (d) f(n)= 0 , for all n

25. If $I(n) = \left[\frac{1}{n}\right]$, then for all n, the value of $I(n)log(n)$ equals
    (a) I(n)
    (b) 1
    (c) 0
    (d) log(n)

26. $\lim_{x\to\infty} \frac{\vartheta(x)}{x}$ is
    (a) 0 (b) 1 (c) ∞ (d) e
27. The correct order of magnitude of $\pi(n)$ is
    (a) $\frac{n}{\log n}$ (b) n log n (c) $\frac{\log n}{n}$ (d) log n
28. Theorems relating to different weighted averages of same functions are called
    (a) Prime number theorem (b) Abel's identity (c) Tauberian theorem (d) None of these
29. $\sum_{p\leq x} \frac{\log p}{p}$ is
    (a) log x + O(1) (b) x log x + O(x) (c) x log x + O(1) (d) x log x- x + O (log x)
30. $\lim_{x\to\infty} (\frac{\psi(x)}{x} - \frac{\vartheta(x)}{x})$ is
    (a) 1 (b) $\frac{(\log x)^2}{2\sqrt{x}\log 2}$ (c) 0 (d) None of these

31. The relation $\sum_{y<n\leq x} a(n)f(n) = A(x)f(x) - A(y)f(y) - \int_y^x A(t)f'(t)\,dt$ where a(n) is an arithmetical function with $A(x) = \sum_{n\leq x} a(n)$ is known as

(a) Euler summation formula (b) Abel's identity (c) Prime number theorem (d) None of these

32. $\psi(x)$ is equivalent to which of the following

(a) $\sum_{p\leq x} \log p$ (b) $\lim_{n\to\infty} \frac{\log x}{x}$ (c) $\sum_{n\leq x} a(n)$ (d) $\sum_{n\leq x} \Lambda(n)$

33. $\vartheta(x)$ is equivalent to which of the following .

(a) $\sum_{p\leq x} \log p$ (b) $\lim_{n\to\infty} \frac{\log x}{x}$ (c) $\sum_{n\leq x} a(n)$ (d) $\sum_{n\leq x} \Lambda(n)$

34. The relation $\lim_{x\to\infty} \frac{\pi(x)\log x}{x}$ = 1 is known as

(a) Prime number theorem (b) Abel's identity (c) Tauberian theorem (d) None of these

35. $\pi(x)\log x - \int_2^x \frac{\pi(t)}{t}\,dt =$ ----------

(a) $\pi(x)$ (b) $\psi(x)$ (c) $\vartheta(x)$ (d) $\frac{P_n}{n}$

36. $\frac{\vartheta(x)}{\log x} + \int_2^x \frac{\vartheta(t)}{log^2 t}dt =$ ------------

(a) $\pi(x)$ (b) $\psi(x)$ (c) $\vartheta(x)$ (d) $\frac{P_n}{n}$

37. Which of the following is not the equivalent form of prime number theorem

(a) $\lim_{x\to\infty} \frac{\pi(x)\log x}{x}$ =1 (b) $\lim_{x\to\infty} \frac{\vartheta(x)}{x} = 1$ (c) $\lim_{x\to\infty} \frac{\psi(x)}{x} = 1$ (d) $\lim_{n\to\infty} \frac{P_n}{n} = 1$

38. $\sum_{n\leq x} \frac{\Lambda(n)}{n} =$ ---------------

(a) x log x + O(1) (b) log x + O(1) (c) x log x + O(x) (d) log x + O(x)

39. $\sum_{p\leq x} \frac{\log p}{p} =$ ------------

(a) log x + O(1) (b) log x + O(x) (c) x log x + O(1) (d) x log x + O(x)

40. $\sum_{n\leq x} \psi(\frac{x}{n}) =$ ---------------

(a) x log x – x + O(log x) (b) x log x + O (x) (c) x log x + O(log x) (d) None of these

41 . $\sum_{n\leq x} \vartheta(\frac{x}{n}) =$ -----------------------

(a) x log x + O(1) (b) x log x + O (x) (c) x log x + O(log x) (d) None of these

42. The relation $\log\log x + A + O(\frac{1}{\log x})$ is equivalent to which of the following where A is a constant and $x \geq 2$

(a) $\sum_{p\leq x} \frac{\log p}{p}$ (b) $\sum_{n\leq x} \frac{\Lambda(n)}{n}$ $\sum_{p\leq x} \frac{1}{p}$ (d) $\sum_{n\leq x} \psi(\frac{x}{n})$

43. Which of the following is the lower bound for the n 'th prime p_n

(a) $\frac{1}{6}\frac{n}{\log n}$ (b) 6 $\frac{n}{\log n}$ (c) $\frac{1}{6}$ $n\log n$ (d) 12 ($n\log n + n\log\frac{12}{e}$)

44. Which of the following is the upper bound for the n 'th prime p_n

(a) $\frac{1}{6}\frac{n}{\log n}$ (b) 6 $\frac{n}{\log n}$ (c) $\frac{1}{6}$ $n\log n$ (d) 12 ($n\log n + n\log\frac{12}{e}$)

45. The theorem stating $(p-1)! \equiv -1 \ (mod\ p)$ is known as

(a) Lagranges theorem (b) Little Fermat theorem (c) Wilson's theorem (d) Wolstenholmes theorem

46. The value of Fermat number F_3 is

(a)17 (b) 154 (c) 235 (d)257

47. Number of solutions of linear congruence $ax \equiv b \ (mod\ m)$ for (a, m)=1 is

(a) 0 (b)1 (c) 2 (d) 3

48. The value of congruence satisfying the congruence $3x \equiv 2 \pmod{5}$

(a) 2 (b)1 (c)4

(d)3

49.The unit digit of 7^{2022} is

(a) 1 (b) 2 (c) 7 (d) 9

50. If X= 48 , Y=15 then the value of k such that $X(\mathrm{mod}\,Y) \equiv (X+KY) \bmod Y$

(a) No such k exists

(b) k is any positive integer

(c) k is any negative integer

(d) k is any integer

51. What is the value of $9^6 \pmod{7}$

(a) 7 (b) 3 (c)5 (d) 1

52. What is the remainder when $2^{20}+3^{30}+4^{40}+5^{50}+11^7$ is divided by 7

(a) 2450 (b) 36 (c) 0 (d) 3

53.If p is an odd prime number then according to Fermats theorem

(a) $2^{P-1} - 2$ is divisible by p

(b) $2^{P-1} -1$ is divisible by p

(c) $2^P - 2$ is not divisible by p

(d) $2^{P} - 2$ is divisible by p

54. The general solution of $x \equiv 5 \pmod{25}$ and $x \equiv 32 \pmod{23}$

(a) 800+55k for k ε z

(b) 55-800k for k ε z

(c) 55+575k for k ε z

(d) 800-55k for k ε z

55. Solution of the following congruence using Chinese Remainder theorem is $x \equiv 3 \bmod (9)$, $x \equiv 7 \bmod (13)$

(a) $x \equiv 107 \pmod{117}$

(b) $x \equiv 103 \pmod{117}$

(c) $x \equiv 111 \pmod{117}$

(d) $x \equiv 105 \pmod{117}$

56. Let k be the order of a mod(n) then $a^b \equiv 1 \bmod (n)$

(a) k divides a
(b) k divides b
(c) k divides n
(d) k divides 1

57. The value of x using Chinese remainder theorem if x≡2(mod3),x≡2(mod4),x≡1(mod5) is a simultaneous system of linear congruences

(a) 60 (b) 146 (c) 47 (d) 256

58. If a and b are any positive integers and a≡b (mod n) then

(a) gcd(a,n) = gcd(b,n)
(b) b divides a-n
(c) f(x)≡1 mod(n)
(d) a and b leave different nonnegative remainder when divided by n

59. The the remainder when 3^{31} is divided by 7 is

(a) 5 (b) 16 (c) 18 (d) 3

60. Which one of the following is correct

(a) 28≡10(mod 6)
(b) 4≡7(mod 3)
(c) 17≡13(mod5)
(d) 8≡13(mod2)

61. Let a,b,c and n are integers ten which of the following is correct

(a) a≡ a(modn)
(b) a≡ b(modn) then b≡ a(modn)
(c) a≡ b(modn) then b≡ c (modn) then a≡ c(modn)
(d) All the above

62. Let a≡ b(modn) and c≡ d(modn) then which of the following is incorrecct

(a) a+c≡ b+d (modn)
(b) a-c≡ b-d (modn)
(c) ac≡bd(modn)
(d) $\frac{a}{c}\equiv\frac{b}{d}$mod(n)

63. The remainder when 41^{60} is divided by 7 is

(a) 2 (b) 1 (c)3 (d) 4

64. If a≡b (mod n_1) and a≡ b(mod n_2) iff a≡ b(mod n) then

(a) n= gcd (n_1, n_2)
(b) n=lcm (n_1, n_2)

(c) n= gcd (a, b)
(d) n= lcm (a,b)

65. Number of incongruent solutions of 35x≡14 (mod21) is
(a) 7 (b) 6 (c) 5 (d) 0

66. Which of the following has a unique incongruent solutions
(a) 8x≡14(mod24)
(b) 20x≡14(mod15)
(c) 12x≡14(mod2)
(d) 15x≡14(mod8)

67.5x≡2(mod3) then
(a) x≡2(mod3)
(b) x≡1 (mod3)
(c) x≡2 (mod2)
(d) x≡1 (mod2)

68. The system of linear congruences ax+by≡ r(modn) and cx+dy≡s(modn) has a unique solution modulo n if
(a) gcd(ad-bc,n) =1
(b) gcd(ad+bc, n) =1
(c) gcd(ab+cd,n) =1
(d) gcd(ab-cd,n) =1

69.If ac≡ bc(mod n) and if gcd(n,c)=d then
(a) a≡b mod($\frac{n}{d}$)
(b)$\frac{a}{d}$≡$\frac{b}{d}$(modn)
(c) a≡ b(mod n)
(d) None of the above

70.Which of the following is reduced residue system modulo 5
(a) {1,2,3,4}
(b){5,7,8,10}
(c) {2,10,25,7}
(d) None of the above

71.The number of solutions of x^2≡ 1(mod 8) is

(a) 1
(b) 2
(c) 0
(d) 4

72. Solution of 5x≡3(mod24) is
(a) 14(mod24)
(b) 15(mod24)
(c) 8 (mod24)
(d) 10(mo24)

73. Solutions of $x^2 \equiv 4\ (mod 7)$ are
a) 2 & 5
b) 2 &6
c) 1 & 2
d) 2 & 4

74. Solutions of $x^2 \equiv 3\ (mod 11)$ are
a) 1 & 3
b) 5 &6
c) 2 & 3
d) No solution

75. The Congruence $x^2 \equiv n\ (mod\ p)$ represents
a) $n\bar{R}p$
b) n is a quadratic non residue mod p
c) n is a quadratic residue mod p
d) None of these

76. Which of the following are equivalent
i. x^2 is not congruent to n mod p
ii. $n\bar{R}p$
iii. n is a quadratic residue mod p
iv. n is a quadratic non residue mod p

a) (i), (ii) & (iv)
b) (i), (iii) & (iv)
c) (ii) & (iii)
d) (i) & (iii)

77. Quadratic residues mod 11 are
a) 2, 6, 7, 8, 10
b) 1, 3, 4, 5, 9
c) 1, 2, 3, 4, 5
d) 6, 7, 8, 9, 10

78. Quadratic non residues mod 11 are
a) 2, 6, 7, 8, 10
b) 1, 3, 4, 5, 9

c) 1, 2, 3, 4, 5
d) 6, 7, 8, 9, 10

79. Quadratic residues mod 13 are
a) 1, 3, 4, 9, 10, 12
b) 2, 5, 6, 7, 8, 11
c) 1, 2, 3, 4, 5, 6
d) 6, 7, 8, 9, 10, 11

80. Quadratic non residues mod 13 are
a) 1, 3, 4, 9, 10, 12
b) 2, 5, 6, 7, 8, 11
c) 1, 2, 3, 4, 5, 6
d) 6, 7, 8, 9, 10, 11

81. The Legendre's symbol (n|p) = ...
a) $\begin{cases} 1 & if\ nRp \\ -1 & if\ n\bar{R}p \end{cases}$
b) $\begin{cases} -1 & if\ nRp \\ 1 & if\ n\bar{R}p \end{cases}$
c) $\begin{cases} 1 & if\ nRp \\ 0 & if\ n\bar{R}p \end{cases}$
d) None of the above

82. The Legendre's symbol (n|p) = ... if $n \equiv o\ (mod\ p)$
a) 0
b) 1
c) -1
d) None of the above

83. The Legendre's symbol (1|p) = ...
a) 0
b) $(-1)^p$
c) -1
d) 1

84. The Legendre's symbol (m^2|p) = ...
a) 0
b) $(-1)^p$
c) -1
d) 1

85. Which of the following equation represents Euler's criterion.
a) (n|p) ≡ $n^{p-1}\ (mod\ p)$
b) (n|p) ≡ $n^{\frac{p-1}{2}}\ (mod\ p)$
c) (n|p) ≡ $n^{\frac{p}{2}}\ (mod\ p)$
d) (n|p) ≡ $n^{\frac{p+1}{2}}\ (mod\ p)$

86. Which of the following is not true about Legendre's symbol (n|p)
a) Completely multiplicative
b) Doesn't vanish when p divides n
c) Periodic with period p and vanish when p divides n

d) It is the quadratic character mod p

87. By Gauss' Lemma, we have the relation (n|p) = $(-1)^m$, where n is not congruent to 0 mod p and m is

a) Number of least positive residues mod p in {n, 2n, 3n, ..., $\frac{(p-1)}{2}$n} which exceed $\frac{p}{2}$
b) Number of residues mod p in {n, 2n, 3n, ..., $\frac{(p-1)}{2}$n} which exceed $\frac{p}{2}$
c) Number of least positive residues mod p in {n, 2n, 3n, ..., $\frac{(p-1)}{2}$n} which does not exceed $\frac{p}{2}$
d) None of the above

88. For every odd prime (-1|p) = ……… if $p \equiv 1 \ (mod\ 4)$.

a) 1
b) -1
c) 0
d) None of the above

89. For every odd prime (-1|p) = ……… if $p \equiv 3 \ (mod\ 4)$.

a) 1
b) -1
c) 0
d) None of the above

90. For every odd prime (2|p) = ……… if $p \equiv \pm 1 \ (mod\ 8)$.

a) 1
b) -1
c) 0
d) None of the above

91. For every odd prime (2|p) = ……… if $p \equiv \pm 3 \ (mod\ 8)$.

a) 1
b) -1
c) 0
d) None of the above

92. If p and q are distinct odd primes then which of the following represents quadratic reciprocity law

a) $(p|q)(q|p) = (-1)^{\frac{(p+q)(q-1)}{4}}$
b) $(p|q) = \begin{cases} -(q|p) & if\ p \equiv q \equiv 3(mod\ 4) \\ (q|p) & if\ p \equiv q \equiv 1(mod\ 4) \end{cases}$
c) $(p|q) = \begin{cases} (q|p) & if\ p \equiv q \equiv 3(mod\ 4) \\ -(q|p) & if\ p \equiv q \equiv 1(mod\ 4) \end{cases}$
d) None of the above

93. The exponent of $a\ modulo\ m$ is …………………

a. the smallest positive integer f such that $a^f \equiv 1(mod\ m)$
b . the smallest integer f such that $a^f \equiv 1(mod\ m)$
c. the greatest integer f such that $a^f \equiv 1(mod\ m)$
d. None of the above

94. Let $m \geq 1$ and $(a, m) = 1$ and $f = exp_m(a)$. Then which of the following is not true.
a. the numbers $1, a, a^2, \dots, a^{f-1}$ are incongruent modulo m.
b. the numbers $1, a, a^2, \dots, a^{f-1}$ are congruent modulo m.
c. $a^f \equiv 1(mod\ m)$
d. $a, a^2, \dots, a^f$ are incongruent modulo m.

95. Which of the following is not a complete residue system for mod m.
a. $\{0,1,2,3, \dots, m-1\}$
b. $\{1,2,3, \dots, m\}$
c. $\{m+1, m+2, \dots, 2m\}$
d. $\{m+1, m+2, \dots, 2m-1\}$
96. Which of the following is not true.
a. $\emptyset(m) \leq exp_m(a)$
b. $exp_m(a) \leq \emptyset(m)$
c. $exp_m(a) | \emptyset(m)$
d. None of the above
97. Primitive root for $m = 1$ is
a. 1
b. 0
c. 2
d. No primitive root
98. Primitive root for $m = 2$ is
a. 1
b. 2
c. 3
d. 4
99.Let x be an odd integer and $\alpha \geq 3$. Then $x^{\frac{\emptyset(2^\alpha)}{2}} \equiv 1\ (mod\ 2^\alpha)$ represents
a. there exists primitive roots $mod\ 2^\alpha$
b. there exist no primitive roots $mod\ 2^\alpha$
c. there exist no primitive roots $mod\ 2p^\alpha$, where p is a prime
d. None of the above
100. If g is a primitive root mod p, where p is an odd prime, then ……………………………
a. $g^2, g^4, \dots, g^{p-1}$ are the quadratic residues mod p
b. $g^2, g^4, \dots, g^{p-1}$ are the quadratic non residues mod p
c. $g, g^3, \dots, g^{p-2}$ are the quadratic residues mod p
d. None of the above.

IV SEMESTER MSc MATHEMATICS
ANALYTIC NUMBER THEORY MCQ ANSWER KEY

1. d
2. a
3. d
4. a
5. b
6. b
7. c
8. a
9. c
10. a
11. b
12. d
13. b
14. a
15. a
16. d
17. c
18. b
19. c
20. c
21. a
22. d
23. b
24. c
25. c
26. b
27. a
28. c
29. a
30. c
31. b
32. d
33. a
34. a
35. c
36. a
37. d
38. b
39. a
40. a
41. b
42. c
43. c
44. d
45. c

46. d
47. b
48. d
49. d
50. d
51.d
52. c
53.d
54. c
55. c
56.b
57.b
58. a
59. d
60. d
61. d
62. d
63. b
64.b
65.a
66.d
67.b
68. a
69. a
70. a
71. d
72. b
73.a
74. b
75.c
76. a
77.b
78.a
79. a
80. b
81. a
82. a
83. d
84.d
85.b
86.b
87. a
88.a
89.b
90.a
91.b
92. b
93.a

94. b
95. d
96. a
97. a
98. c
99. b
100. a