ME010401 - SPECTRAL THEORY

MCQ for private students

1. Which of the following space is reflexive?

   A. l^1

   B. l^2

   C. l^∞

   D. $L^1[a,b]$

2. Which of the following space is not reflexive?

   A. $\mathbb{R}^n$

   B. l^2

   C. l^1

   D. l^3

3. Consider the normed space X and the functional g_x defined by $g_x(f) = f(x),\ \forall f \in X'$. Then which of the following is true?

   A. $||g_x|| = ||x||$

   B. $||g_x|| < ||x||$

   C. $||g_x|| > ||x||$

   D. None of these

4. Consider $\mathbb{R}$ with usual metric. Which of the following set is nowhere dense in $\mathbb{R}$?

   A. $\mathbb{Q}$

   B. $\mathbb{N}$

   C. $(1, 2)$

   D. None of these.

5. Which of the following is a set of first category?

A. $\mathbb{R}$

B. $\mathbb{C}$

C. $\mathbb{R}^n$

D. $\mathbb{Z}$

6. Which of the following is a set of second category?

A. $\mathbb{N}$

B. $\mathbb{Z}$

C. $\mathbb{Q}$

D. $\mathbb{R}$

7. Let X be a finite dimensional normed space. Which of the following is false?

A. strong convergence $\Longrightarrow$ weak convergence

B. weak convergence $\Longrightarrow$ strong convergence

C. weak convergence $\not\Rightarrow$ strong convergence

D. strong convergence $\Longleftrightarrow$ weak convergence

8. X is a reflexive space and $C : X \to X''$ be the canonical mapping. Then which of the following is true?

A. C is bijective

B. C is linear

C. C is isometric

D. all of the above

9. Which of the following statement is false?

A. If X is reflexive, then it is complete.

B. If X is complete, then it is reflexive.

C. Every Hilbert space is reflexive.

D. Every finite dimensional normed space is reflexive.

10. Let X and Y be normed spaces and $\{T_n\}$ be a uniformly operator convergent sequence in $B(X,Y)$ with limit T. Which of the following statement is true?

   A. $||T_n - T|| \to 0$

   B. $||T_n x - Tx|| \to 0, \ \ \forall x \in X$

   C. $|f(T_n x) - f(Tx)| \to 0, \ \ \forall x \in X$ and $\forall f \in Y'$

   D. All the above

11. Let X and Y be normed spaces and $\{T_n\}$ be a strongly operator convergent sequence in $B(X,Y)$ with limit T. Find the statement which is not always true.

   A. $||T_n - T|| \to 0$

   B. $||T_n x - Tx|| \to 0, \ \ \forall x \in X$

   C. $|f(T_n x) - f(Tx)| \to 0, \ \ \forall x \in X$ and $\forall f \in Y'$

   D. $||f(T_n x) - f(Tx)|| \to 0, \ \ \forall x \in X$ and $\forall f \in Y'$

12. In a normed space X it is given that $x_n \rightharpoonup x$. Which of the following is true?

   A. The sequence $\{||x_n||\}$ is bounded

   B. For every element f of a total subset $M \subset X'$, $f(x_n) \to f(x)$

   C. If X is a Hilbert space then $\langle x_n, z\rangle \to \langle x, z\rangle, \ \ \forall z \in X$

   D. All the above

13. Let $T_n \in B(X,Y)$ where X is a Banach space and Y is a normed space. Let $\{T_n\}$ is strongly operator convergent with limit T. Which of the following statement is false?

   A. $T \in B(X,Y)$

   B. The sequence $\{||T_n||\}$ is bounded

C. The sequence $\{T_n x\}$ is cauchy in Y for every x in a total subset M of X

D. None of these

14. Which of the following mapping from $\mathbb{R}$ to $\mathbb{R}$ is an open mapping?

A. $f(x) = \sin x$

B. $f(x) = \cos x$

C. $f(x) = k$, where k is a constant

D. $f(x) = 2x$

15. Let X and Y be Banach spaces, $T \in B(X, Y)$ and T is bijective. Which of the following statement is false?

A. T is open

B. T^{-1} is bounded

C. T is continuous

D. T^{-1} is continuous

16. Let X and Y be Banach spaces, $T \in B(X, Y)$ and $B_n = B(0, \frac{1}{2^n})$. Then which of the following is true?

A. $\overline{T(B_1)}$ contains an open ball in Y

B. $\overline{T(B_n)}$ contains an open ball about $0 \in Y$

C. $T(B_0)$ contains an open ball about $0 \in Y$

D. All the above

17. Let X and Y be normed spaces and $T : X \to Y$ be a closed linear operator. Then which of the following statement is always true?

A. $\{(x, Tx) : x \in X\}$ is closed in $X \times Y$

B. T is bounded

C. T is continuous

D. T maps closed sets onto closed sets

18. Let X and Y be normed spaces and $T : X \to Y$ be a closed linear operator. Then which of the following statement is false?

A. $\{(x, Tx) : x \in X\}$ is closed in $X \times Y$

B. T maps closed sets to closed sets

C. If $x_n \to x$, $x_n \in D(T)$ and $Tx_n \to y$, then $x \in D(T)$ and $y = Tx$

D. If T is bounded and Y is complete, then $D(T)$ is closed in X.

19. Let $X = C[0, 1]$ and T be the differential operator $x \to x'$. Which of the following is true?

A. T is closed and bounded

B. T is bounded but not closed

C. T is closed but unbounded

D. T is neither closed nor bounded

20. Consider the mapping $T : \mathbb{R}^2 \to \mathbb{R}$ defined by $T(x, y) = x$. Which of the following is false?

A. T is linear and bounded

B. T continuous

C. T is open

D. None of these

21. Consider the mapping $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x, 0)$. Which of the following is true?

A. T is linear and bounded

B. T is open

C. T is $1 - 1$ and onto

D. T^{-1} is continuous

22. Let $S_n, T_n \in B(X, Y)$ are strongly operator convergent sequences with limits S and T respectively. What is the strong limit of $\{S_n + T_n\}$?

A. Not exist

B. $S + T$

C. Exist but cannot be determined

D. ST

23. Let X and Y be normed spaces. $T \in B(X, Y)$, $x_n \rightharpoonup x$ in X, Then what is the weak limit of $\{Tx_n\}$ in Y?

A. Tx

B. $f(Tx)$ for some $f \in Y'$

C. Does not exist

D. Not unique

24. Let $\{x_n\}$ and $\{y_n\}$ are sequences in a normed space X such that $x_n \rightharpoonup x$ and $y_n \rightharpoonup y$. Thenwhat is the weak limit of $\{x_n + y_n\}$ in X?

A. Not exist

B. $x + y$

C. 0

D. Cannot be determined uniquely

25. Let $x_n \rightharpoonup x$ in a normed space X. Then which of the following statement is false?

A. $f(x_n) \rightarrow f(x), \ \forall f \in X'$

B. $(\alpha x_n) \rightharpoonup \alpha x, \ \forall \alpha \in K$

C. $||x_n - x|| \rightarrow 0$

D. x is unique

26. A fixed point of a mapping $T : X \rightarrow X$ of a set X is defined by

A. $Tx = -x$

B. $Tx = x$

C. $Tx = x^2$

D. $Tx = 0$

27. Fixed points of the mapping $Tx = x^2$ on $\mathbb{R}$ is

A. 1 and -1

B. 0 and 1

C. 0 and -1

D. None of these.

28. Consider the mapping $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x, 0)$. The fixed points of T are

A. $(0, 0)$ only

B. $(0, 1)$ only

C. $(1, 0)$ only

D. $(0, 0)$ and $(1, 1)$

29. Banach Fixed point theorem is stated as

A. Let $X \neq \phi$ be a complete metric space and let $T : X \to X$ be a contraction on X. Then T has at most one fixed point.

B. Let $X \neq \phi$ be a metric space and let $T : X \to X$ be a contraction on X. Then T has precisely one fixed point.

C. Let $X \neq \phi$ be a metric space and let $T : X \to X$ be a contraction on X. Then T has at most one fixed point.

D. Let $X \neq \phi$ be a complete metric space and let $T : X \to X$ be a contraction on X. Then T has precisely one fixed point.

30. A mapping is called contraction on a metric space if

A. There is a positive real number $\alpha < 1$ such that $\forall x, y \in X$, $d(Tx, Ty) \leq \alpha d(x, y)$

B. There is a positive real number $\alpha < 1$ such that $\forall x, y \in X$, $d(Tx, Ty) \geq \alpha d(x, y)$

C. There is a positive real number $\alpha > 1$ such that $\forall x, y \in X$, $d(Tx, Ty) \leq \alpha d(x, y)$

D. There is a positive real number $\alpha > 1$ such that $\forall x, y \in X$, $d(Tx, Ty) \geq \alpha d(x, y)$

31. The fixed points of the mapping $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x, 0)$ are

A. Only $(0, 0)$.

B. All points on the y axis.

C. All points on the x axis.

D. None of these.

32. A contraction T on a complete metric space

A. Is always Continuous

B. Has precisely one fixed point

C. Both (A) and (B)

D. None of these

33. The eigen values of the matrix $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$ are

A. 6 and 1

B. 2 and 3

C. -2 and -3

D. -6 and -1

34. The characteristic equation of the matrix $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$ is

A. $\lambda^2 + 7\lambda - 6 = 0$

B. $\lambda^2 + 7\lambda + 6 = 0$

C. $\lambda^2 - 7\lambda + 6 = 0$

D. $\lambda^2 - 7\lambda - 6 = 0$

35. Which one is not true for spectral radius of an operator $T \in B(X, X)$ on a complex Banach space X.

A. $r_\sigma(T) \leq ||T||$

B. $r_\sigma(T) = \sup\limits_{\lambda \in \sigma(T)} |\lambda|$

C. $r_\sigma(T) = \lim\limits_{n \to \infty} \sqrt[n]{||T^n||}$

D. The radius of the largest closed disk centred at the origin of the complex plane and containing $\sigma(T)$.

36. Which of the following statement is true.
Eigen vectors $x_1, x_2, \cdots x_n$ corresponding to different eigen values $\lambda_1, \lambda_2, \cdots \lambda_n$ of a linear operator T on a vector space X

A. Constitute a linearly independent set

B. Constitute a linearly dependent set

C. Constitute an idempotent set

D. None of these

37. Let $X \neq \phi$ be a complex normed space and λ be a regular value of the linear operator $T : D(T) \to X$ with $D(T) \subseteq X$. Then which of the following is false?

A. $R_\lambda(T)$ is exist

B. $R_\lambda(T)$ is unbounded

C. $R_\lambda(T)$ is defined on a set which is dense in X

D. $R_\lambda(T)$ bounded

38. The set of all λ such that $R_\lambda(T)$ exists, $R_\lambda(T)$ is defined on a set which is dense in X and $R_\lambda(T)$ is unbounded is called

A. Point spectrum

B. Continuous spectrum

C. Residual spectrum

D. Resolvent set

39. The set of all λ such that $R_\lambda(T)$ does not exist is called

A. Point spectrum

B. Continuous spectrum

C. Residual spectrum

D. Resolvent set

40. The set of all λ such that $R_\lambda(T)$ exists, $R_\lambda(T)$ is defined on a set which is not dense in X is called

A. Point spectrum

B. Continuous spectrum

C. Residual spectrum

D. Resolvent set

41. The set of all regular values of a linear operator is called

A. Point spectrum

B. Continuous spectrum

C. Residual spectrum

D. Resolvent set

42. An $n \times n$ matrix B is said to be similar to an $n \times n$ matrix A if

A. there exists a non-singular matrix C such that $B = C^{-1}AC$

B. there exists a singular matrix C such that $B = C^{-1}AC$

C. there exists a Hermitian matrix C such that $B = C^{-1}AC$

D. there exists a skew-Hermitian matrix C such that $B = C^{-1}AC$

43. The spectrum of a linear operator on a finite dimensional space is

A. Point spectrum

B. Continuous spectrum

C. Residual spectrum

D. All the above

44. Let $T : l^2 \to l^2$ be a linear operator defined by $T(x_1, x_2, \cdots) = (0, x_1, x_2, \cdots)$. Then spectral value of T is

A. $\{1\}$

B. $\{0\}$

C. $\{0, 1\}$

D. $\{-1\}$

45. The spectrum of a bounded linear operator T on a complex Banach space X is

A. Open and lies in the disk given by $|\lambda| < ||T||$

B. Open and compact

C. Closed, compact and lies in the disk given by $|\lambda| \leq ||T||$

D. None of these

46. The resolvent set of a bounded linear operator T on a complex Banach space X is

A. Open

B. Closed

C. Compact

D. None of these

47. Resolvent of a linear operator T on a complex Banach space X does not satisfy

   A. $R_\mu - R_\lambda = (\mu - \lambda)R_\mu R_\lambda$, where $\lambda, \mu \in \rho(T)$

   B. If $ST = TS$ then $SR_\lambda = R_\lambda S$, where $S \in B(X, X)$

   C. $R_\lambda R_\mu = R_\mu R_\lambda$, where $\lambda, \mu \in \rho(T)$

   D. If $\lambda \in \rho(S) \cap \rho(T)$ then $R_\lambda(S) - R_\lambda(T) = R_\lambda(S)(S - T)R_\lambda(T)$ where $S \in B(X, X)$

48. Let X be a complex Banach space and $T : X \to X$ a linear operator and . If T is closed or bounded, then

   A. $R_\lambda(T)$ is defined on the whole space X

   B. $R_\lambda(T)$ is bounded

   C. $R_\lambda(T)$ is defined on the whole space X and bounded

   D. None of these

49. A bounded linear operator T on a Banach space is said to be idempotent if

   A. $T^2 = T$

   B. $T^2 = T^{-1}$

   C. $T^2 = 0$

   D. $T^2 = I$

50. Which of the following statement is false?

   A. A linear operator on a finite dimensional complex normed space $X \neq \{0\}$ has at least one eigen value

   B. Similar matrices have same eigen values

   C. Matrices representing a given linear operator $T : X \to X$ on a finite dimensional normed space X relative to various bases for X have different eigenvalues.

D. Eigenvectors corresponding to different eigenvalues of a linear operator T on a vector space X constitute a linearly independent set

51. A normed algebra is a normed space which is an algebra with identity e satisfying:

A. $\|xy\| \leq \|x\| + \|y\|$ for all $x, y \in A$

B. $\|xy\| = \|x\| \, \|y\|$ for all $x, y \in A$

C. $\|xy\| \leq \|x\| \, \|y\|$ for all $x, y \in A$

D. $\|xy\| = \|x\| + \|y\|$ for all $x, y \in A$

52. Which of the following is not a commutative Banach Algebra:

A. Real line $\mathbf{R}$

B. Complex plane $\mathbf{C}$

C. $B(X, X)$ with $dimX = n$

D. $C[a, b]$

53. Let A be a complex Banach algebra with identity e. If $x \in A$, then $e - x$ is invertible when:

A. $\|x\| \leq 1$

B. $\|x\| < 1$

C. $\|x\| \geq 1$

D. $\|x\| > 1$

54. Let A be a complex Banach algebra with identity e. The set of all invertible elements of A form:

A. a closed subset of A

B. an open subset of A

C. a compact subset of A

D. a complete subspace of A

55. Let A be a complex Banach algebra with identity e. Then for any $x \in A$

A. the spectrum $\sigma(x)$ is bounded

B. the spectrum $\sigma(x)$ is closed

C. the spectrum $\sigma(x)$ is compact

D. all the above

56. The identity operator $I : X \to X$ defined on an infinite dimensional space X is:

A. compact

B. not compact

C. not bounded

D. none of the above

57. Let X and Y be normed spaces and $T : X \to Y$ a bounded linear operator, then :

A. T is compact, if $dim T(X) < \infty$

B. T is compact, if $dim X < \infty$

C. T is compact, if $dim X < \infty$ and $dim T(X) < \infty$

D. all the above

58. Let (T_n) be a sequence of compact linear operators from a normed space X into a Banach space Y, then :

A. T is compact, if (T_n) is strongly operator convergent to T

B. T is compact, if (T_n) is uniformly operator convergent to T

C. T is compact, if (T_n) is weakly operator convergent to T

D. all the above

59. The zero operator on any normed space is :

A. not bounded

B. not compact

C. compact

D. none of the above

60. Let X and Y be normed spaces and $T : X \to Y$ a compact linear operator, then :

A. (Tx_n) is strongly convergent in Y if (x_n) is weakly convergent in X

B. (Tx_n) is strongly convergent in Y if (x_n) is strongly convergent in X

C. $)(Tx_n)$ is weakly convergent in Y if (x_n) is weakly convergent in X

D. all the above

61. Let B be a subset of a metric space X :

A. if B is relatively compact, then B is totally bounded

B. if B is totally bounded, then B is separable

C. if B is totally bounded and X is complete, then B is relatively compact

D. all the above

62. Let B be a subset of a metric space X :

A. if B is relatively compact, then B is totally bounded

B. if B is totally bounded, then B is separable

C. if B is totally bounded, then B is bounded

D. all the above

63. Let X and Y be normed spaces and $T : X \to Y$ a compact linear operator, then :

A. the range $R(T)$ is separable

B. $T(X)$ is separable

C. the range $R(T)$ has a countable dense subset

D. all the above

64. Let X and Y be normed spaces and $T : X \rightarrow Y$ a compact linear operator. If X' and Y' are the dual spaces of X and Y then :

A. the adjoint operator $T^{\times} : Y' \rightarrow X'$ is compact

B. the adjoint operator $T^{\times} : Y' \rightarrow X'$ is bounded

C. the adjoint operator $T^{\times} : Y' \rightarrow X'$ exists

D. all the above

65. Let X be a normed space and $T : X \rightarrow X$ a compact linear operator, then the set of eigenvalues of X is:

A. empty

B. finite

C. countable

D. uncountable

66. Let X be a normed space and $T : X \rightarrow X$ a compact linear operator, then the set of eigenvalues of X has:

A. no accumulation point

B. only possible accumulation point 0

C. only possible accumulation point 1

D. none of the above

67. Let X be a normed space and $T : X \rightarrow X$ a compact linear operator, then for $\lambda \neq 0$ the eigenspace of T is:

A. countable

B. uncountable

C. finite

D. empty

68. Let X be a normed space and $T : X \to X$ a compact linear operator, then every spectral value $\lambda \neq 0$:

A. is an eigenvalue

B. is a regular value

C. is in residual spectrum

D. is in continuous spectrum

69. Let X be an infinite dimensional normed space and $T : X \to X$ a compact linear operator, then:

A. $0 \notin \sigma(T)$

B. $0 \in \sigma(T)$

C. $\sigma(T) = \phi$

D. none of the above

70. Let A be a complex Banach algebra with identity e. Which of the following statement is true?

A. The set of all λ in the complex plane such that $x - \lambda e$ is not invertible is the spectrum of x.

B. The set of all λ in the complex plane such that $x - \lambda e$ is invertible is the spectrum of x.

C. The set of all λ in the complex plane such that $x - \lambda e$ is not invertible is the resolvant set of x.

D. none of the above

71. Let A be a complex Banach algebra with identity e. If $x \in A$ then, the spectral radius $r_\sigma(x)$ satisfies:

A. $r_\sigma(x) < \|x\|$

B. $r_\sigma(x) \leq \|x\|$

C. $r_\sigma(x) > \|x\|$

D. $r_\sigma(x) \geq \|x\|$

72. Let X and Y be normed spaces and $T : X \to Y$ a compact linear operator, then :

A. $\overline{T(M)}$ is compact, for any bounded set $M \subseteq X$

B. $T(M)$ is compact, for any bounded set $M \subseteq X$

C. $T(M)$ is relatively compact, for any set $M \subseteq X$

D. all the above

73. Let X and Y be normed spaces and $T : X \to Y$ a compact linear operator, then :

A. T is bounded

B. T is continous

C. T is bounded and continuous

D. T is bounded but need not be continuous

74. Let X and Y be normed spaces, $T : X \to Y$ a compact linear operator and $U = \{x \in X : \|x\| = 1\}$, then :

A. $\overline{T(U)}$ is not compact

B. $\overline{T(U)}$ is compact

C. $T(U)$ is not relatively compact

D. none of the above

75. Let X be a normed space and $T : X \to X$ a compact linear operator, then for $\lambda \neq 0$:

A. the nullspaces of $T_\lambda, {T_\lambda}^2, {T_\lambda}^3, ...$ are closed

B. the nullspaces of $T_\lambda, {T_\lambda}^2, {T_\lambda}^3, ...$ are infinite dimensional

C. the ranges of $T_\lambda, {T_\lambda}^2, {T_\lambda}^3, ...$ are closed

D. none of the above

76. Let H be a Hilbert space. The bounded linear operator $T : H \to H$ satisfying the condition $\langle Tx, y\rangle = \langle x, T^*y\rangle$ for all $x, y \in H$ is called

A. Hilbert-adjoint operator

B. Adjoint operator

C. Self-adjoint operator

D. Normal operator

77. A bounded linear operator $T : H \to H$ on a complex Hilbert space H is self-adjoint if and only if

A. $T = T^{-1}$

B. $TT^* = T^*T$

C. $T = T^*$

D. $TT^* = 0$

78. Let $T : H \to H$ be a bounded self-adjoint linear operator on a complex Hilbert space H. Then the eigenvalues of T (if they exist) are

A. 0 or 1

B. imaginary

C. real

D. None of these

79. Let $T : H \to H$ be a bounded self-adjoint linear operator on a complex Hilbert space H. Then a number λ belongs to the resolvent set $\rho(T)$ of T if and only if there exists a $c > 0$ such that for every $x \in H$,

A. $\|T_\lambda x\| \leq c\|x\|$

B. $\|T_\lambda x\| \geq c\|x\|$

C. $\|T_\lambda x\| = c\|x\|$

D. $\|T_\lambda x\| \neq c\|x\|$

80. The spectrum $\sigma(T)$ of a bounded self-adjoint linear operator $T : H \to H$ on a complex Hilbert space H is

A. real

B. imaginary

C. empty

D. $\{0, 1\}$

81. The residual spectrum $\sigma_r(T)$ of a bounded self-adjoint linear operator $T : H \to H$ on a complex Hilbert space H is

A. empty

B. $\{0\}$

C. $\{0, 1\}$

D. imaginary

82. A bounded self-adjoint linear operator $T : H \to H$ on a Hilbert space H is said to be positive if

A. $\langle Tx, x\rangle = 0 \quad \forall x \in H$

B. $\langle Tx, x\rangle < 0 \quad \forall x \in H$

C. $\langle Tx, x\rangle > 0 \quad \forall x \in H$

D. $\langle Tx, x\rangle \geq 0 \quad \forall x \in H$

83. A bounded linear operator $P : H \to H$ on a Hilbert space H is a projection if and only if

A. P is self-adjoint

B. P is idempotent

C. P is self-adjoint and idempotent

D. P is normal

84. For any projection P on a Hilbert space H, which one of the following is correct?

A. $\langle Px, x \rangle = \|Px\|^2 \quad \forall x \in H$

B. $P \geq 0$

C. $\|P\| \leq 1$

D. All the above

85. Two closed subspaces Y and V of a Hilbert space H are orthogonal if and only if the corresponding projections P_Y and P_V satisfy

A. $P_Y P_V = I$

B. $P_Y P_V \neq I$

C. $P_Y P_V \neq 0$

D. $P_Y P_V = 0$

86. Let P_1 and P_2 be projections on a Hilbert space H. Then which one of the following is false?

A. The product $P = P_1 P_2$ is a projection on H if and only if the projections P_1 and P_2 commute

B. The sum $P = P_1 + P_2$ is a projection on H if and only if $Y_1 = P_1(H)$ and $Y_2 = P_2(H)$ are orthogonal

C. The difference $P = P_2 - P_1$ is a projection on H if and only if $Y_1 \subset Y_2$, where $Y_1 = P_1(H)$ and $Y_2 = P_2(H)$

D. None of the above

87. Let P_1 and P_2 be projections on a Hilbert space H and let $Y_1 = P_1(H)$ and $Y_2 = P_2(H)$. If the product $P = P_1 P_2$ is a projection, P projects H onto

A. $Y_2 \cap Y_1^{\perp}$

B. $Y_1 \cap Y_2$

C. $Y_1 \oplus Y_2$

D. $Y_1 \cup Y_2$

88. Let P_1 and P_2 be projections on a Hilbert space H and let $Y_1 = P_1(H)$ and $Y_2 = P_2(H)$. If the sum $P = P_1 + P_2$ is a projection, P projects H onto

A. $Y_2 \cap Y_1^{\perp}$

B. $Y_1 \cap Y_2$

C. $Y_1 \oplus Y_2$

D. $Y_1 \cup Y_2$

89. Let P_1 and P_2 be projections on a Hilbert space H and let $Y_1 = P_1(H)$ and $Y_2 = P_2(H)$. If the difference $P = P_2 - P_1$ is a projection, P projects H onto

A. $Y_2 \cap Y_1^{\perp}$

B. $Y_1 \cap Y_2$

C. $Y_1 \oplus Y_2$

D. $Y_1 \cup Y_2$

90. Let P be a projection on a Hilbert space H and let $Y = P(H)$. Which one of the following is false?

A. $Y^{\perp}$ is the null space of P

B. $I - P$ also projects H onto Y

C. $P|_Y$ is the identity operator on Y

D. Y is a closed subspace of H

91. Which one of the following is correct?

A. The sum of positive operators is positive.

B. The product of positive operators is positive.

C. The difference of positive operators is positive.

D. None of these

92. For any bounded self-adjoint linear operator T on a complex Hilbert space H, $\|T\| =$

A. $\inf_{\|x\| \neq 1} |\langle Tx, x\rangle|$

B. $\sup_{\|x\|=1} |\langle Tx, x\rangle|$

C. $\sup_{\|x\|=1} \langle Tx, x\rangle$

D. $\inf_{\|x\|=1} \langle Tx, x\rangle$

93. Which of the following is not a spectral value of a bounded self-adjoint linear operator $T : H \to H$ on a complex Hilbert space H.

A. i

B. $m = \inf_{\|x\|=1} \langle Tx, x\rangle$

C. $M = \sup_{\|x\|=1} \langle Tx, x\rangle$

D. None of these

94. Let $T : H \to H$ be a bounded self-adjoint linear operator on a complex Hilbert space H. The eigenvectors corresponding to different eigenvalues of T are

A. Normal

B. Orthogonal

C. In the same eigenspace

D. None of these

95. Let $T : H \to H$ be a bounded self-adjoint linear operator on a Hilbert space H and let $m = \inf_{\|x\|=1} \langle Tx, x\rangle$ and $M = \sup_{\|x\|=1} \langle Tx, x\rangle$. Then which one of the following is correct?

A. $\rho(T) \subset [m, M]$

B. $\rho(T) \subset (m, M)$

C. $\sigma(T) \subset [m, M]$

D. $\sigma(T) \subset (m, M)$

96. If $T : H \to H$ is a self-adjoint linear operator on a complex Hilbert space H, then $\langle Tx, x \rangle$ is

A. 0

B. 1

C. Real

D. Imaginary

97. If T_1 and T_2 are bounded self-adjoint linear operators on a complex Hilbert space H, which of the following is equivalent to $T_1 \leq T_2$

A. $0 \leq T_2 - T_1$

B. $T_2 - T_1$ is positive

C. $\langle T_1 x, x \rangle \leq \langle T_2 x, x \rangle \quad \forall x \in H$

D. All of these

98. If the product ST of two positive operators S and T on a Hilbert space H is positive, if

A. S and T are bounded

B. S and T commute

C. S and T are continuous

D. None of these

99. Let H is a Hilbert space and Y is a closed subspace of H. If $Y^{\perp}$ denotes the orthogonal complement of Y, which of the following is true?

A. $H = Y \cup Y^{\perp}$

B. $H = Y \cap Y^{\perp}$

C. $H = Y \oplus Y^{\perp}$

D. None of these

100. P is an idempotent operator if and only if

A. $P = P^{-1}$

B. $\langle Px, x \rangle$ is real

C. $P^{*} = P$

D. $P^{2} = P$

ME010401 - SPECTRAL THEORY

Answer Key

1. B
2. C
3. A
4. B
5. D
6. D
7. C
8. D
9. B
10. D
11. A
12. D
13. D
14. D
15. C
16. D
17. A
18. B
19. C
20. D

21. A
22. B
23. A
24. B
25. C
26. B
27. B
28. A
29. D
30. A
31. C
32. C
33. A
34. C
35. D
36. A
37. B
38. B
39. A
40. C
41. D
42. A

43. A

44. B

45. C

46. A

47. D

48. C

49. A

50. C

51. C

52. C

53. B

54. B

55. D

56. B

57. D

58. B

59. C

60. D

61. D

62. D

63. D

64. D

65. C

66. B

67. C

68. A

69. B

70. A

71. B

72. A

73. C

74. B

75. C

76. A

77. C

78. C

79. B

80. A

81. A

82. D

83. C

84. D

85. D

86. D

87. B
88. C
89. A
90. B
91. A
92. B
93. A
94. B
95. C
96. C
97. D
98. B
99. C
100. D

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