ME010304 - FUNCTIONAL ANALYSIS

MCQ for private students

1. Which of the following is triangle inequality?

   A. $d(x, y) = d(y, x)$
   B. $d(x, y) \geq d(x, z) + d(z, y)$
   C. $d(x, y) \leq d(x, z) + d(z, y)$
   D. $d(x, y) \leq d(x, z) + d(z, w)$

2. Which of the following is the usual metric on $\mathbb{R}^2$?. Where $x = (x_1, y_1)$ and $y = (x_2, y_2)$.

   A. $d(x, y) = (x_1 - x_2)^2 + (y_1 - y_2)^2$
   B. $d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$
   C. $d(x, y) = |x_1 - x_2|^2 + |y_1 - y_2|^2$
   D. $d(x, y) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

3. Let M be a nonempty subset of a metric space (X, d). Then $x \in \bar{M}$ if and only if:

   A. For every sequence $\{x_n\}$ in M such that $x_n \to x$.
   B. There is a sequence $\{x_n\}$ in M such that $x_n \to x$.
   C. $x \in M$
   D. None of these.

4. Which of the following space is not complete?

   A. Set of rationals $\mathbb{Q}$
   B. Function space $C[a, b]$
   C. Euclidean space $\mathbb{R}^n$
   D. Sequence space l^∞

5. Which of the following subset of $\mathbb{R}$ is complete?

A. $(0, 1]$

B. $[0, 1)$

C. $[0, 1]$

D. $(0, 1)$

6. Which of the following space is not separable?

   A. $\mathbb{R}$

   B. l^∞

   C. $\mathbb{C}$

   D. l^p

7. Which of the following is not true?

   A. Every Cauchy sequence in a metric space is a convergent sequence.

   B. Every Cauchy sequence in a complete metric space is a convergent sequence.

   C. Every convergent sequence in a metric space is a Cauchy sequence.

   D. The complex plane is a complete metric space.

8. Which of the following is a vector space?

   A. $\mathbb{Q}$ over $\mathbb{R}$.

   B. $\mathbb{R}$ over $\mathbb{C}$.

   C. $\mathbb{R}$ over $\mathbb{Q}$.

   D. None of these.

9. Which of the following is not a vector space?

   A. $\mathbb{R}$ over $\mathbb{Q}$.

   B. $\mathbb{Q}$ over $\mathbb{R}$.

   C. $\mathbb{C}$ over $\mathbb{R}$.

   D. $\mathbb{C}$ over $\mathbb{C}$.

10. What will be the dimension of the vector space $\mathbb{R}^n$ over $\mathbb{R}$?.

    A. $2n$

    B. n^2

    C. $n-1$

    D. n

11. Which of the following subsets of $\mathbb{R}^2$ are linearly dependent?
    $A = \{(1,2),(2,3),(3,4)\}$
    $B = \{(1,-1),(1,1),(0,2)\}$
    $C = \{(1,-1),(1,1)\}$
    $D = \{(1,1),(2,4),(3,9)\}$

    A. Only A, B and D.

    B. Only C

    C. Only A and D.

    D. Only B and D.

12. Let W_1 and W_2 are subspaces of a vector space X, then which of the following is not a subspace X?

    A. $\text{span}(W_1 \cup W_2)$

    B. $W_1 \cup W_2$

    C. $W_1 \cap W_2$

    D. $\text{span}(W_1)$

13. Which of the following is a subspace of $\mathbb{R}^3$?. Where $(x, y, z) \in \mathbb{R}^3$.

    A. $x - y + z - 2 = 0$

    B. $-5x + 7y - 8z = 3$

    C. $2x - y + 3z = 0$

    D. $z - y - 1 = 0$

14. Which of the following is not a property of norm ?

   A. $||x|| \geq 0$
   B. $||\alpha x|| = \alpha\, ||x||$
   C. $||x + y|| \leq ||x|| + ||y||$
   D. $||x|| = 0 \Longleftrightarrow x = 0$

15. Let X be a normed space and $x, y \in X$. Which of the following is not true?

   A. $||x - y|| \leq ||x|| - ||y||$
   B. $||x - y|| \leq ||x|| + ||y||$
   C. $||x + y|| \leq ||x|| + ||y||$
   D. $|\,||x|| - ||y||\,| \leq ||x - y||$

16. A complete normed space is known as a:

   A. Hilbert space
   B. Separable space
   C. Compact space
   D. Banach space

17. Which of the following statement is true?

   A. Every normed space is a metric space.
   B. Every normed space is a Banach space.
   C. Every metric space is a normed space.
   D. None of these.

18. The metric induced by the norm is:

   A. $d(x, y) = ||x + y||$
   B. $d(x, y) = ||x - y||$

C. $d(x,y) = ||x-y||^2$

D. $d(x,y) = \sqrt{||x-y||}$

19. Let X be a normed space and d be the metric induced by the norm. Then for $x, y \in X$ which of the following is true?

A. $d(6x+3y, 3x-y) = d(3x, 2y)$

B. $d(6x-3y, 3x+y) = d(2y, 3x)$

C. $d(6x-3y, 3x-y) = d(2y, 3x)$

D. $d(6x+3y, 3x+y) = d(3x, 4y)$

20. Let X be a normed space and d is the metric induced by the norm, then for any scalar α which of the following is true?

A. $d(\alpha x, \alpha y) = d(x,y)$

B. $d(\alpha x, \alpha y) = \alpha\, d(x,y)$

C. $d(\alpha x, \alpha y) = |\alpha|^2\, d(x,y)$

D. $d(\alpha x, \alpha y) = |\alpha|\, d(x,y)$

21. Which of the following statement is false?

A. Every finite dimensional subspace Y of a normed space X is closed in X.

B. Every finite dimensional subspace Y of a normed space X is complete X.

C. Every subspace Y of a normed space X is closed in X.

D. Every complete subspace Y of a Banach space X is closed.

22. Consider the following statements:

i Every subspace of a Banach space is complete.

ii Every normed space can be identified as a dense subspace of a Banach space.

A. Both (i) and (ii) are true.

B. Neither (i) nor (ii) are true.

C. Only (i) is true.

D. Only (ii) is true.

23. When we can say that a norm $||.||$ on a vector space X is equivalent to a norm $||.||_o$ on X ?

A. $\forall\ a, b > 0$ such that $\exists x, y \in X$, $a||x||_o \leq ||x|| \leq b||x||_o$

B. $\exists\ a, b > 0$ such that $\forall x, y \in X$, $a||x||_o \leq ||x|| \leq b||x||_o$

C. $\exists\ a, b > 0$ such that $\forall x, y \in X$, $a||x||_o \leq ||x|| \leq b||x||$

D. $\exists\ a, b > 0$ such that $\forall x, y \in X$, $a||x|| \leq ||x|| \leq b||x||_o$

24. Consider the following statements:

i Every closed and bounded subset of a metric space is compact.

ii Every subset M of a finite dimensional normed space X is compact if and only if M is closed in X.

A. Both (i) and (ii) are true.

B. Neither (i) nor (ii) are true.

C. Only (i) is true.

D. Only (ii) is true.

25. Every finite dimensional subspace of a normed space is:

A. Complete

B. Finite

C. Open

D. Compact

26. Consider $T : \mathbb{R}^2 \to \mathbb{R}$. Which of the following is a linear map?

A. $T(x, y) = x^2 y$

B. $T(x, y) = xy$

C. $T(x, y) = 2x - y$

D. $T(x, y) = x^2$

27. Consider the following statements:

i Every finite dimensional normed space is complete.

ii Two norms defined on a finite dimensional vector space are equivalent.

A. Both (i) and (ii) are true.

B. Neither (i) nor (ii) are true.

C. Only (i) is true.

D. Only (ii) is true.

28. Let T be a linear operator:

i The range of T is not a vector space.

ii The null space of T is a vector space.

A. Both (i) and (ii) are true.

B. Neither (i) nor (ii) are true.

C. Only (i) is true.

D. Only (ii) is true.

29. The rank of a linear operator is:

A. dim $\mathcal{N}(T)$

B. dim $\mathcal{D}(T)$

C. dim $\mathcal{R}(T)$

D. None of these.

30. A linear operator T is injective, then

A. $x_1 \neq x_2 \implies Tx_1 = Tx_2$

B. $\mathcal{N}(T) = \{0\}$

C. $\mathcal{N}(T) \neq \{0\}$

D. None of these.

31. Let X and Y be vector spaces. Let $T : X \to Y$ be a linear operator. Then the inverse $T^{-1} : \mathcal{R}(T) \to X$ exists if and only if:

A. T is bounded.

B. T is continuous.

C. T is compact.

D. $\mathcal{N}(T) = \{0\}$

32. Let X and Y are normed spaces and $T : X \to Y$ be a bounded linear operator:

A. T maps open subsets of X into open subsets of Y.

B. T maps every bounded sets in X onto unbounded sets in Y.

C. T maps every bounded sets in X onto bounded sets in Y.

D. None of these

33. Let X and Y be normed spaces. The linear operator $T : X \to Y$ is said to be bounded if there is a real number c such that $\forall x \in X$,

A. $||Tx|| \geq c||x||$

B. $||Tx|| \leq c||x||$

C. $||x|| \leq c||Tx||$

D. None of these.

34. Let T_1 and T_2 are bounded linear operators defined on a normed space X. Then which of the following is true?

A. $||T_1T_2|| \geq ||T_1||||T_2||$

B. $||T_1T_2|| = ||T_1||||T_2||$

C. $||T_1T_2|| \leq ||T_1||||T_2||$

D. None of these.

35. Which of the following operator is not bounded?

A. The differential operator defined on the normed space of all polynomials on $[0, 1]$

B. The identity operator on a normed space.

C. The zero operator on a normed space.

D. None of these.

36. Consider the following statements:

i Every linear operator defined on a finite dimensional normed space is unbounded.

ii If a linear operator T is continuous then it is bounded.

A. Both (i) and (ii) are true.

B. Neither (i) nor (ii) are true.

C. Only (i) is true.

D. Only (ii) is true.

37. Pick the incorrect statement:

A. If a linear operator T is continuous then it is bounded.

B. If a linear operator T is bounded then it is continuous.

C. The null space of a bounded linear operator is open.

D. Every linear operator defined on a finite dimensional normed space is bounded.

38. Let T be a bounded linear operator. Then $x_n \to x$ [where $x_n, x \in \mathcal{D}(T)$] implies:

A. $\{TX_n\}$ need not onverges.

B. $Tx_n \to Tx$

C. $Tx_n \to Tx_0$

D. None of these.

39. Let T be a bounded linear operator defined on a normed space X. Let $x, y \in \mathcal{N}(T)$ then

A. $x + y \in \mathcal{N}(T)$.

B. $x - y \in \mathcal{N}(T)$.

C. $-x + y \in \mathcal{N}(T)$.

D. All are true.

40. T_1 and T_2 are two invertible linear operators on a vector space X. Pick the correct one:

A. $T_1 - T_2$ is invertible.

B. T_1T_2 is invertible.

C. $T_1 + T_2$ is invertible.

D. None of these.

41. Let f be a bounded linear functional on a normed space X then $||f||$ is:

A. $\sup\limits_{||x||=1} |f(x)|$

B. $\sup\limits_{x \neq 0} |f(x)|$

C. $\sup\limits_{x \neq 0} ||f(x)||$

D. None of these.

42. A linear functional f with domain $\mathcal{D}(f)$ in a normed space X is bounded if and only if:

A. X is complete.

B. X is finite dimensional.

C. X is compact.

D. None of these.

43. An isomorphism T of a metric space $X = (X, d)$ onto a metric space $\tilde{X} = (\tilde{X}, \tilde{d})$ is a bijective mapping such that, for all $x, y \in X$

A. $d(x, y) = d(Tx, Ty)$

B. $\tilde{d}(x, y) = \tilde{d}(Tx, Ty)$

C. $\tilde{d}(x, y) = d(Tx, Ty)$

D. $d(x, y) = \tilde{d}(Tx, Ty)$

44. A vector space X is said to be algebraically reflexive if:

A. Canonical mapping $C : X \to X^{**}$ is surjective.

B. Canonical mapping $C : X \to X^{**}$ is injective.

C. Canonical mapping $C : X \to X^{**}$ is not surjective.

D. Canonical mapping $C : X \to X^{**}$ is not injective.

45. What will be the dual space of $\mathbb{R}^3$?

A. $\mathbb{C}^3$

B. $\mathbb{R}^3$

C. l^3

D. None of these.

46. Let X be an $n-$ dimensional vector space. What will be the dimension of X^{**}?

A. n^2

B. $2n$

C. n^{n^2}

D. n

47. Consider the following statements:

    i A finite dimensional vector space is algebraically reflexive.

    ii Let X be a finite dimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0 \;\; \forall f \in X^*$, then $x_0 = 0$.

A. Both (i) and (ii) are true.

B. Neither (i) nor (ii) are true.

C. Only (i) is true.

D. Only (ii) is true.

48. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (x, y, -x - y)$. What will be the matrix which represents T with respect to the standard basis of $\mathbb{R}^3$?

A. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 0 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{pmatrix}$

D. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix}$

49. What will be the dual space of l^1 ?

A. l^2

B. $\mathbb{R}$

C. l^∞

D. $\mathbb{C}$

50. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T(x, y, z) = (x+y-z, 2x+3y+5x, x-y+7z)$. What will be the nullity of T ?

A. 3

B. 0

C. 1

D. 2

51. What will be the basis for the null space of the functional f defined on $\mathbb{R}^3$ by $f(a, b, c) = a + b - c$?

A. $\{(-1, 1, 1), (1, 0, 1)\}$

B. $\{(-1, 1, 0), (-1, 0, 1)\}$

C. $\{(1, 1, 0), (1, 0, 1)\}$

D. $\{(-1, 1, 0), (1, 0, 1)\}$

52. The set of all bounded linear operators from a normed space X into a normed space Y is a banach space if:

A. Y is a banach space.

B. X is a banach space.

C. Both X and Y must be banach.

D. None of these.“

53. Dual space of l^p is:

A. l^∞

B. l^q, where $\frac{1}{p} - \frac{1}{q} = 1$.

C. l^q, where $\frac{1}{p} + \frac{1}{q} = 1$.

D. l^p

54. Consider the following statements:

i The dual space X' of a normed space X is a Banach space.

ii The dual space X' of a normed space X is a Hilbert space.

A. Both (i) and (ii) are true.

B. Neither (i) nor (ii) are true.

C. Only (i) is true.

D. Only (ii) is true.

55. Which of the following is not a property of inner product in general?

A. $\langle x, y\rangle = \overline{\langle y, x\rangle}$

B. $\langle \alpha x, y\rangle = \bar{\alpha}\langle x, y\rangle$

C. $\langle x, x\rangle = 0 \Longleftrightarrow x = 0$

D. $\langle x, \alpha y\rangle = \bar{\alpha}\langle x, y\rangle$

56. A Hilbert space is:

A. Complete normed space.

B. Complete metric space.

C. Complete vector space.

D. Complete inner product space.

57. Consider the following statements:

i All Banach spaces are Hilbert spaces.

ii All Hilbert spaces are metric spaces.

A. Both (i) and (ii) are true.

B. Neither (i) nor (ii) are true.

C. Only (i) is true.

D. Only (ii) is true.

58. Which of the following statement is false?

A. All normed spaces are inner product spaces.

B. All Banach spaces are metric spaces.

C. All Hilbert spaces are Topological spaces.

D. All inner product spaces are normed spaces.

59. What will be the metric induced by an inner product?

A. $d(x, y) = \langle x - y, x - y \rangle$

B. $d(x, y) = \sqrt{\langle x - y, x - y \rangle}$

C. $d(x, y) = \sqrt{\langle x, y \rangle}$

D. None of these.

60. Which of the following is known as parallelogram equality?

A. $||x + y||^2 + ||x - y||^2 = 2||x||^2 + ||y||^2$

B. $||x + y||^2 - ||x - y||^2 = 2(||x||^2 + ||y||^2)$

C. $||x + y||^2 + ||x - y||^2 = 2(||x||^2 + ||y||^2)$

D. $||x + y||^2 + ||x - y||^2 = 2(||x||^2 - ||y||^2)$

61. Let X be an inner product space and $x, y \in X$ such that $||x|| = 3$, $||y|| = 2$ and $||x - y|| = 1$, then what will be $||x + y||$?

A. 5

B. 4

C. 3

D. 1

62. Let X be an inner product space and $x, y \in X$ such that $||x + y|| = 10$ and $||x - y|| = 6$, then what will be $\langle x, y \rangle$?

A. 4

B. 16

C. 64

D. 136

63. Two vectors x and y in an inner product space are orthogonal if

A. $\langle x-y, x-y\rangle = 0$

B. $\langle x-y, x-y\rangle \neq 0$

C. $\langle x, y\rangle \neq 0$

D. $\langle x, y\rangle = 0$

64. Which of the following space is a Hilbert space?

A. l^2

B. l^3

C. $c[a, b]$

D. None of these.

65. If $x \perp y$ then:

A. $||x+y||^2 = 2(||x||^2 + ||y||^2)$

B. $||x+y||^2 = 2||x||^2 + ||y||^2$

C. $||x+y||^2 = ||x||^2 + ||y||^2$

D. $||x-y||^2 = ||x||^2 - ||y||^2$

66. Which of the following is the Schwarz inequality?

A. $|\langle x, y\rangle| \geq ||x||\,||y||$

B. $|\langle x, y\rangle| \leq ||x||\,||y||$

C. $||x+y|| \leq ||x|| + ||y||$

D. $||x+y|| \geq ||x|| + ||y||$

67. In an inner product space X, which of the following is not true in general?

A. $x_n \to x$ and $y_n \to y$ implies $\langle x_n, y_n \rangle \to \langle x, y \rangle$

B. $|\langle x, y \rangle| \leq ||x|| \, ||y||$

C. $||x + y||^2 + ||x - y||^2 = 2(||x||^2 + ||y||^2)$

D. $||x + y||^2 = ||x||^2 + ||y||^2$

68. Let X be an inner product space and $x \in X$, α be any scalar, then $||\alpha x||^2$ is:

A. $|\alpha|^2 \, ||x||^2$

B. $|\alpha| \, ||x||^2$

C. $\alpha^2 \, ||x||^2$

D. Not defined.

69. An isomorphism T of an inner product space $(X, \langle \ \rangle)$ onto an inner product space $(\tilde{X}, \langle \ \rangle_1)$ over the same field is a bijective linear operator $T : X \to \tilde{X}$ such that $\forall x, y \in X$:

A. $\langle x, y \rangle_1 = \langle Tx, Ty \rangle$

B. $\langle x, Ty \rangle = \langle Tx, y \rangle_1$

C. $\langle Tx, y \rangle_1 = \langle x, Ty \rangle_1$

D. $\langle x, y \rangle = \langle Tx, Ty \rangle_1$

70. Let M be a subspace of a Hilbert space H. Then which of the following is false?

A. $M^{\perp}$ is a subspace of H.

B. $M \subset M^{\perp\perp}$

C. $M \cap M^{\perp} = \{0\}$

D. $M = M^{\perp\perp}$

71. If M is a closed subspace of a Hilbert space H. Then which of the following is false?

A. $M = M^{\perp\perp}$

B. M is complete.

C. Span of M is dense in H.

D. $H = M \oplus M^{\perp}$

72. A subset M of a Hilbert space H is total in H if and only if:

A. $M^{\perp} \neq \{0\}$

B. $M^{\perp} = \{0\}$

C. Span of $M^{\perp}$ is dense in H.

D. $\overline{M} = H$

73. Which of the following statement is true?

A. Every closed ball in a normed space is convex.

B. Subspace of a normed space is not convex.

C. Union of two convex sets is convex.

D. Intersection of two convex sets is not convex.

74. Let Y be any closed subspace of a Hilbert space H and $P : H \to Y$ be the orthogonal projection H onto Y. Then which of the following statement is false?

A. P is idempotent.

B. P is a bounded operator.

C. P is linear but not bounded.

D. $\mathcal{N}(P) = Y^{\perp}$

75. Let M be an orthonormal set in an inner product space X. Then which of the following statement is false?

A. M is orthogonal.

B. All elements in M have norm 1.

C. M is linearly independent.

D. M is linearly dependent.

76. Which of the following set is orthonormal in $\mathbb{R}^3$?

A. $\{(1,0,0),(0,1,0),(0,0,0)\}$

B. $\{(-1,0,0),(0,-1,0),(0,0,-1)\}$

C. $\{(1,0,0),(0,1,0),(1,0,1)\}$

D. $\{(-1,0,1),(1,-1,0),(0,1,-1)\}$

77. Consider the inner product space $\mathbb{R}^3$ with usual inner product. Let $\{(1,0,0),(0,\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}),z\}$ is an orthonormal set in $\mathbb{R}^3$. Then what will be z?

A. $(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$

B. $(0,\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$

C. $(0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$

D. $(0,-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}})$

78. What will be the distance between two orthonormal vectors in an inner product space?

A. 1

B. $\sqrt{3}$

C. $\sqrt{2}$

D. 2

79. Let M be a subspace of an inner product space X. What will be the $\{0\}^{\perp}$?

A. X

B. $M^{\perp}$

C. M

D. 0

80. Let $\{e_k\}$ be an orthonormal sequence in an inner product space X. Then $\forall x \in X$, the Bessel inequality is:

A. $\sum_{k=1}^{\infty} |\langle x, e_k \rangle|^2 \geq ||x||^2$
B. $\sum_{k=1}^{\infty} |\langle x, e_k \rangle| \leq ||x||$
C. $\sum_{k=1}^{\infty} |\langle x, e_k \rangle| \geq ||x||$
D. $\sum_{k=1}^{\infty} |\langle x, e_k \rangle|^2 \leq ||x||^2$

81. Let $\{e_k\}$ be an orthonormal sequence in a Hilbert space H. Then the series $\sum_{k=1}^{\infty} \alpha_k e_k$ converges if and only if:

A. $\sum_{k=1}^{\infty} |\alpha_k|^3$ converges.
B. $\sum_{k=1}^{\infty} |\alpha_k|$ converges.
C. $\sum_{k=1}^{\infty} |\alpha_k|^4$ converges.
D. None of these.

82. An orthonormal set $M = \{e_k\}$ in a Hilbert space H is total in H if and only if $\forall x \in H$:

A. $\sum_k |\langle x, e_k \rangle|^2 < ||x||^2$
B. $\sum_k |\langle x, e_k \rangle|^2 = ||x||^2$
C. $\sum_k |\langle x, e_k \rangle|^2 > ||x||^2$
D. None of these.

83. Hilbert dimension is:

A. Cardinality of an orthonormal set in a Hilbert space $H \neq \{0\}$.
B. Cardinality of a orthogonal set in a Hilbert space $H \neq \{0\}$.
C. Cardinality of a total orthonormal set in a Hilbert space $H \neq \{0\}$.
D. Cardinality of a total subset in a Hilbert space $H \neq \{0\}$.

84. Let H be a Hilbert space.

i H is separable if it has a countable dense subset.

ii If H is separable, every orthonormal set in H is countable.

A. Both (i) and (ii) are true.

B. Neither (i) nor (ii) are true.

C. Only (i) is true.

D. Only (ii) is true.

85. Let f be a bounded linear functional defined on $\mathbb{R}^2$ by $f(x, y) = 2x+3y$. What will be the unique term in $\mathbb{R}^2$, when f is represented in terms of the inner product?

A. $(3, 2)$

B. $(2, 3)$

C. $(-2, 3)$

D. $(2, -3)$

86. Which of the following is not a property of sesquilinear form?

A. $h(x + y, z) = h(x, z) + h(y, z)$

B. $h(\alpha x, y) = \alpha\, h(x, y)$

C. $h(x, \beta y) = \bar{\beta}\, h(x, y)$

D. $h(x, \beta y) = \beta\, h(x, y)$

87. Let H_1 and H_2 are Hilbert spaces and $T : H_1 \to H_2$ be a bounded linear operator and T^* be its Hilbert - adjoint operator. Then $\forall x \in H_1$ and $y \in H_2$, which of the following is true?

A. $\langle T^*x,\, y\rangle = \langle x,\, Ty\rangle$

B. $\langle Ty,\, x\rangle = \langle y,\, T^*x\rangle$

C. $\langle T^*y,\, x\rangle = \langle y,\, Tx\rangle$

D. $\langle T^*y,\, x\rangle = \langle Ty,\, x\rangle$

88. Let H_1 and H_2 are Hilbert spaces and $T : H_1 \to H_2$ be a bounded linear operator and T^* be its Hilbert - adjoint operator. Which of the following is false?

A. $(\alpha T)^* = \alpha T^*$

B. $(T^*)^* = T$

C. $||T|| = ||T^*||$

D. $||T^*T|| = ||T||^2$

89. A bounded linear operator $T : H \to H$ on a Hilbert space H is normal if:

A. $T^* = T$

B. $T^* T = T T^*$

C. $T^* = T^{-1}$

D. $(T^*)^* = T$

90. Let $T : H \to H$ be a bounded linear operator on a Hilbert space H. Which of the following is true?

A. If T is normal then T is self adjoint.

B. If T is normal then T is unitary.

C. If T is unitary then T^{-1} is not unitary.

D. If T is unitary then T is normal.

91. Let S and T are two bounded self adjoint linear operators on a Hilbert space H.

i $\langle Tx, x\rangle$ is real $\forall x \in H$.

ii The product ST is always self adjoint.

A. Both (i) and (ii) are true.

B. Neither (i) nor (ii) are true.

C. Only (i) is true.

D. Only (ii) is true.

92. Let S be a unitary operators on a Hilbert space H. Which of the following is false?

A. S^{-1} is not unitary.

B. S is normal.

C. S is isometric.

D. $||S|| = 1$, provided $H \neq \{0\}$

93. Let T be a bounded linear operators on a Hilbert space H. Which of the following operator is not self adjoint?

A. $\frac{1}{2}(T + T^*)$

B. $\frac{1}{2i}(T + T^*)$

C. $\frac{1}{2i}(T - T^*)$

D. None of these.

94. Which of the following is not a partially ordered set?

A. The set of all real numbers with $x \leq y$ mean that x is less than or equal to y.

B. The power set $\mathcal{P}(X)$ of a given set X and $A \leq B$ mean that A is a subset of B.

C. The set of all positive integers and $x \leq y$ mean that x divides y.

D. None of these.

95. Consider the partially ordered set, power set of X with $A \leq B$ mean that A is a subset of B. What will be the maximal element of $\mathcal{P}(X)$?

A. ϕ

B. X

C. No maximal element.

D. None of these.

96. Consider the following statements.

i Every vector space $X \neq \{0\}$ has a Hamel basis.

ii In every Hilbert space $H \neq \{0\}$ there exists a total ortonormal set.

A. Both (i) and (ii) are true.

B. Neither (i) nor (ii) are true.

C. Only (i) is true.

D. Only (ii) is true.

97. Which of the following is a chain?

A. The set of all positive integers and $x \leq y$ mean that x divides y.

B. The power set $\mathcal{P}(X)$ of a given set X and $A \leq B$ mean that A is a subset of B.

C. The set of all real numbers with $x \leq y$ mean that x is less than or equal to y.

D. None of these.

98. For every x in a normed space X:

A. $||x|| = \sup\limits_{f \neq 0} \frac{|f(x)|}{||f||}$

B. $||x|| = \sup\limits_{f \in X',\ f \neq 0} \frac{|f(x)|}{||f||}$

C. $||x|| = \sup\limits_{f \in X',\ f \neq 0} \frac{||f(x)||}{|f|}$

D. $||x|| = \sup\limits_{f \in X'} |f(x)|$

99. Let $T : X \to Y$ be a bounded linear operator, where X and Y are normed spaces. What will be domain of the Adjoint operator $T^{\times}$?

A. X

B. Y

C. X'

D. Y'

100. Which of the following is not true about the Adjoint operator $T^{\times}$?

A. $(\alpha T)^{\times} = \bar{\alpha} T^{\times}$

B. $T^{\times}$ is bounded.

C. $||T|| = ||T^{\times}||$

D. $T^{\times}$ is linear.

ME010304 - FUNCTIONAL ANALYSIS

Answer Key

1. C
2. D
3. B
4. A
5. C
6. B
7. A
8. C
9. B
10. D
11. A
12. B
13. C
14. B
15. A
16. D
17. A
18. B
19. C
20. D

21. C
22. D
23. B
24. B
25. A
26. C
27. A
28. D
29. C
30. B
31. D
32. C
33. B
34. C
35. A
36. D
37. C
38. B
39. D
40. B
41. A
42. D

43. D

44. A

45. B

46. D

47. A

48. A

49. C

50. B

51. D

52. A

53. C

54. C

55. B

56. D

57. D

58. A

59. B

60. C

61. A

62. B

63. D

64. A

65. C

66. B

67. D

68. A

69. D

70. D

71. C

72. B

73. A

74. C

75. D

76. B

77. A

78. C

79. A

80. D

81. D

82. B

83. C

84. A

85. B

86. D

87. C
88. A
89. B
90. D
91. C
92. A
93. B
94. D
95. B
96. A
97. C
98. B
99. D
100. A

‘