PG SEMESTER III-ME010303 :MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

Multiple Choice Questions

Module I

1. Weiestrass Approximation theorem approximates a function to a ....
   (a) Polynomial function
   (b) Cubic function
   (c) Trigonometric function
   (d) None of the above

2. Fourier integral theorem assumes the underlying function to be
   (a) Lebesgue integrable
   (b) Riemann integrable
   (c) Both of the above
   (d) None of the above

3. In order to take a convolution if two functions f and g , both of them should be
   (a) Lebesgue integrable
   (b) Riemann integrable
   (c) Both of the above
   (d) None of the above

4. Convolution of f and g is defined as
   (a) $\int_0^\infty f(t)g(x-t)dt$
   (b) $\int_{-\infty}^\infty f(t)g(x-t)dt$
   (c) $\int_{-\infty}^\infty f'(t)g(x-t)dt$
   (d) $\int_{-\infty}^\infty f(t)g'(x-t)dt$

5. Fourier transform of convolution of two functions is,
   (a) Convolution of Fourier transforms.

(b) Integral of Fourier transforms.

(c) Product of Fourier transforms.

(d) None of the above.

6. $\int_0^1 x^{p-1}(1-x)^{q-1}dx =$

   (a) $\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$

   (b) $\frac{\Gamma(p)+\Gamma(q)}{\Gamma(p+q)}$

   (c) $\frac{\Gamma(p)\Gamma(q)}{\Gamma(pq)}$

   (d) $\frac{\Gamma(p)-\Gamma(q)}{\Gamma(p+q)}$

7. Fourier integral theorem assumes

   (a) f is of bounded variation

   (b) f(x+) and f(x-) exist

   (c) Both f(x+) and f(x-) are Lebesgue integrals

   (d) All of the above

8. In $\int_{-\infty}^{\infty} K(x,y)f(x)dx, K(x,y)$ is known as

   (a) The constant

   (b) The integrant

   (c) The integral

   (d) The Kernel

9. Which of the following expressions are integral transforms of f?
   (i) $\int_{-\infty}^{\infty} e^{-ixy}f(x)dx$ (ii)$\int_0^{\infty} cosxyf(x)dx$ (iii)$\int_0^{\infty} sinxyf(x)dx$ (iv) $\int_0^{\infty} e^{-xy}f(x)dx$

   (a) (i)and (iv)

   (b) (ii) and (iii)

   (c) (i),(ii),(iii) and (iv)

   (d) (ii),(iii) and (iv)

10. Which of the following is the exponential Fourier transform

   (a) $\int_{-\infty}^{\infty} e^{-ixy}f(x)dx$

   (b) $\int_0^{\infty} cosxyf(x)dx$

   (c) $\int_0^{\infty} sinxyf(x)dx$

   (d) $\int_0^{\infty} e^{-xy}f(x)dx$

11. Which of the following is the Fourier cosine transform

(a) $\int_0^\infty e^{-ixy} f(x)dx$
(b) $\int_0^\infty cosxy f(x)dx$
(c) $\int_0^\infty sinxy f(x)dx$
(d) $\int_0^\infty e^{-xy} f(x)dx$

12. Which of the following is the Fourier sine transform

(a) $\int_0^\infty e^{-ixy} f(x)dx$
(b) $\int_0^\infty cosxy f(x)dx$
(c) $\int_0^\infty sinxy f(x)dx$
(d) $\int_0^\infty e^{-xy} f(x)dx$

13. Which of the following is the Laplace transform

(a) $\int_0^\infty e^{-ixy} f(x)dx$
(b) $\int_0^\infty cosxy f(x)dx$
(c) $\int_0^\infty sinxy f(x)dx$
(d) $\int_0^\infty e^{-xy} f(x)dx$

14. Which of the following is the Mellin transform

(a) $\int_0^\infty e^{-ixy} f(x)dx$
(b) $\int_0^\infty cosxy f(x)dx$
(c) $\int_0^\infty sinxy f(x)dx$
(d) $\int_0^\infty x^{y-1} f(x)dx$

15. When we take convolution of two functions, discontinuities of both functions,

(a) Will vanish
(b) Will continue to exist
(c) Type of continuity changes
(d) Merge and becomes one discontinuity.

16. A convolution integral become bounded if

(a) $f \in L^2(R)$
(b) $g \in L^2(R)$
(c) Both a and b.
(d) Neither a nor b.

17. What is the boundedness assumption of the convolution theorem,

(a) Both f and g are continuous

(b) At least one of them is continuous

(c) One of them is a bounded variation

(d) Both are bounded variations

18. A convolution is

 (a) An integral function

 (b) Product of two function

 (c) Composition of two functions

 (d) None of the above

19. Which one the following is the notation for convolution of two functions.

 (a) fog

 (b) $f.g$

 (c) $f * g$

 (d) $\int fg$

20. Let $z(t) = x(t) * y(t)$, where "$*$" denotes convolution. Let c be a positive real-valued constant. Choose the correct expression for z(ct)

 (a) $c\cdot x(ct) * y(ct)$

 (b) $x(ct) * y(ct)$

 (c) $c\cdot x(t) * y(ct)$

 (d) $c\cdot x(ct) * y(t)$

21. According to Fourier integral theorem we have the formula

 (a) $\frac{f(x+)+f(x-)}{2} = lim_{\alpha\to+\infty}\frac{1}{\pi}\int_0^\alpha \left[\int_{-\infty}^{\infty} f(u)cosv(u-x)du\right] dv$

 (b) $\frac{f(x+)+f(x-)}{2} = lim_{\alpha\to+\infty}\frac{1}{2\pi}\int_0^\alpha \left[\int_{-\infty}^{\infty} f(u)cosv(u-x)du\right] dv$

 (c) $\frac{f(x+)+f(x-)}{2} = lim_{\alpha\to+\infty}\pi\int_0^\alpha \left[\int_{-\infty}^{\infty} f(u)cosv(u-x)du\right] dv$

 (d) None of the above

22. An integral operator is always

 (a) a linear operator

 (b) a polynomial operator

 (c) a nonlinear operator

 (d) none of the above

23. We have the Fourier transform $f(x) \sim \frac{a_0}{2} + \Sigma_{n=1}^{\infty}\left(a_n cos(\frac{2\pi nx}{p}) + b_n sin(\frac{2\pi nx}{p})\right)$. Then

(a) $a_n = \frac{2}{p}\int_0^p f(t) cos\frac{2\pi nt}{p} dt$

(b) $a_n = \frac{2}{p}\int_0^p f(t) sin\frac{2\pi nt}{p} dt$

(c) $a_n = \frac{p}{2}\int_0^p f(t) cos\frac{2\pi nt}{p} dt$

(d) $a_n = \frac{p}{2}\int_0^p f(t) sin\frac{2\pi nt}{p} dt$

24. We have the Fourier transform $f(x) \sim \frac{a_0}{2} + \Sigma_{n=1}^{\infty}\left(a_n cos(\frac{2\pi nx}{p}) + b_n sin(\frac{2\pi nx}{p})\right)$. Then

(a) $b_n = \frac{2}{p}\int_0^p f(t) cos\frac{2\pi nt}{p} dt$

(b) $b_n = \frac{2}{p}\int_0^p f(t) sin\frac{2\pi nt}{p} dt$

(c) $b_n = \frac{p}{2}\int_0^p f(t) cos\frac{2\pi nt}{p} dt$

(d) $b_n = \frac{p}{2}\int_0^p f(t) sin\frac{2\pi nt}{p} dt$

25. A Fourier transform generally breaks a function into

(a) sine function

(b) cosine function

(c) a linear combination of (a) and (b)

(d) a sum of (a) and (b)

26. The inversion formula for Fourier Transform is given by

(a) $f(x) = lim_{\alpha \to +\infty} \frac{1}{\pi} \int_{-\alpha}^{+\alpha} g(u) e^{ixu} du$

(b) $f(x) = lim_{\alpha \to +\infty} \pi \int_{-\alpha}^{+\alpha} g(u) e^{ixu} du$

(c) $f(x) = lim_{\alpha \to +\infty} 2\pi \int_{-\alpha}^{+\alpha} g(u) e^{ixu} du$

(d) $f(x) = lim_{\alpha \to +\infty} \frac{1}{2\pi} \int_{-\alpha}^{+\alpha} g(u) e^{ixu} du$

Module 2

27. Pick the correct statement from the following

(a) Existence of all the partial derivatives $\mathbf{D_1f}, \mathbf{D_2f}, \mathbf{D_3f}, ..., \mathbf{D}_n\mathbf{f}$ of a function $\mathbf{f}$ at a particular point doesnot necessarily imply continuity of $\mathbf{f}$ at that point.

(b) The partial derivative describes the rate of change of a function in the direction of each coordinate axis.

(c) The directional derivative is the rate of change of a function in an arbitrary direction

(d) All of the above

28. Let $\mathbf{f}$ be a linear function. Then the directional derivative of $\mathbf{f}$ at the point c in the direction of the unit vector $\mathbf{u}(\mathbf{u} \neq 0)$ is

   (a) $\mathbf{f}(\mathbf{u})$
   (b) 0
   (c) $\mathbf{f}'(\mathbf{u})$
   (d) 1

29. Consider the function $f : R^2 \to R$ given by
$$f(x, y) = \begin{cases} x + y & \text{if } x = 0 \ or \ y = 0 \\ 1 & otherwise \end{cases}$$
Then $D_1 f(0, 0), D_2 f(0, 0)$ is

   (a) 0,0
   (b) 0,1
   (c) 1,1
   (d) x,y

30. The directional derivative of the function f at a point c in the direction of the vector u, where u=0 is

   (a) f(u)
   (b) 0
   (c) $f'(u)$
   (d) 1

31. Choose the correct statement

   (a) If the directional derivative of a function f exists for every direction u, then all the partial derivatives exists.
   (b) If all the partial derivatives of a function f exists, then the directional derivative of f exists for every direction u.
   (c) If the directional derivative of a function f exists at a point c then f is continuous at c.
   (d) If existence of derivative of a function f at a point c doesnot imply the continuity of a function of that point.

32. The total derivative of a linear function is ....

   (a) zero

(b) its derivative

(c) its partial derivative

(d) the function itself

33. If the total derivative of a function f exists, it is ....

(a) unique

(b) a real number

(c) finite

(d) None of the the above

34. If the total derivative of a function f exists, then

(a) it is equal to the directional derivative.

(b) it is unique

(c) it is not unique

(d) both (a) and (b)

35. Cauchy-Reimann equation is

(a) $D_1u(c) = D_1v(c), D_2u(c) = D_2v(c)$

(b) $D_1u(c) = D_2u(c), D_1v(c) = -D_2v(c)$

(c) $D_1u(c) = D_2v(c), D_1v(c) = -D_2u(c)$

(d) $D_1u(c) = D_2v(c), D_1v(c) = D_2u(c)$

36. Let S be an open connected subset and let $f : S \to R^n$ be differentiable at each c in S. If $f^{'}(c) = 0$ for each c in S, then f is

(a) constant

(b) continuous

(c) linear

(d) all of the above

37. Let $f = (f_1, f_2)$ be a function in R^2 which is differentiable at a point c in R^2. Then the Jacobian matrix of f at c is ....

(a) $\begin{bmatrix} D_1f_1(c) & D_2f_2(c) \\ D_1f_2(c) & D_2f_1(c) \end{bmatrix}$

(b) $\begin{bmatrix} D_1f_1(c) & D_2f_1(c) \\ D_1f_2(c) & D_2f_2(c) \end{bmatrix}$

(c) $\begin{bmatrix} D_1f_2(c) & D_2f_2(c) \\ D_1f_1(c) & D_2f_1(c) \end{bmatrix}$

(d) $\begin{bmatrix} D_1f_1(c) & D_1f_2(c) \\ D_2f_1(c) & D_2f_2(c) \end{bmatrix}$

38. Jacobian matrix of the function $f(x, y) = (x^2 - y^2, 2xy)$ at $(1, 1)$ is ....

(a) $\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 2 \\ 2 & -2 \end{bmatrix}$

39. The gradient of the function $f(x, y) = x(x^2 - y^2) - z$ is ....

(a) $(3x^2, -2y, -1)$

(b) $(x^2 - y^2, -xy^2, -z)$

(c) $(x^2 - 2y, 2xy, -1)$

(d) $(3x^2 - y^2, -2xy, -1)$

40. The gradient vector of the function $f(x, y, z) = 2x^2 + 3y^2 + z^2 - 11$ at $(1, 0, 3)$ is ....

(a) $(2, 0, 9)$

(b) $(4, 0, 6)$

(c) $(-7, -11, -5)$

(d) $(4, 6, 11)$

41. The directional derivative of $f(x, y, z) = xyz$ at the point $(-1, 1, 3)$ in the direction of the vector $i - 2j + 2k$ is ....

(a) $7/3$

(b) 7

(c) $-7/3$

(d) -3

42. The directional derivative of the scalar function $f(x, y, z) = x^2 + 2y^2 + z$ at the point $P = (1, 1, 2)$ in the direction of the vector $\vec{a} = 3i - 4j$ is

(a) -4

(b) -2

(c) -1

(d) 1

43. The partial derivatives $D_1 f$ and $D_2 f$ of the function $f(x,y) = x^4 + y^4 - 4x^2y^2$ is

(a) $4x^3, 4y^3$

(b) x^4, y^4

(c) $4x^3 - 4x^2, 4y^3 - 4y^2$

(d) $4x^3 - 8xy^2, 4y^3 - 8x^2y$

44. The partial derivatives $D_1 f$ and $D_2 f$ of the function $f(x,y) = log(x^2 + y^2)$, $(x,y) \neq (0,0)$ is

(a) x^2, y^2

(b) $\frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}$

(c) $\frac{1}{x^2+y^2}, \frac{1}{x^2+y^2}$

(d) $\frac{2}{x^2+y^2}, \frac{2}{x^2+y^2}$

45. If the Cauchy-Reimann equations are true, then the derivative of the function f at a point c is

(a) $f^{'}(c) = D_1 u(c) + iD_1 v(c)$

(b) $f^{'}(c) = D_1 v(c) + iD_1 u(c)$

(c) $f^{'}(c) = D_2 v(c) - iD_2 u(c)$

(d) Both (a) and (c)

46. If $f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4} & \text{if } x \neq 0 \\ 0 & ifx = 0 \end{cases}$

then $f(x,y)$ is

(a) continuous and directional derivative exists at origin.

(b) not continuous at origin but the directional derivative exists at origin.

(c) continuous but directional derivative does not exist at origin.

(d) neither continuous nor directional derivative exists at origin.

47. Jacobian matrix of the function $f(x,y,z) = (e^{xy} + z, x^2yz, xyz)$

(a) $\begin{bmatrix} ye^{xy} & 2xyz & yz \\ xe^{xy} & x^2z & xz \\ 1 & x^2y & xy \end{bmatrix}$

(b) $\begin{bmatrix} ye^{xy} & xe^{xy} & 1 \\ 2xyz & x^2z & x^2y \\ yz & xz & xy \end{bmatrix}$

(c) $\begin{bmatrix} e^{xy} & e^{xy} & 1 \\ yz & x^2z & x^2y \\ yz & xz & xy \end{bmatrix}$

(d) $\begin{bmatrix} ye^{xy} & xe^{xy} & 1 \\ x^2y & 2xyz & x^2z \\ xy & yz & xz \end{bmatrix}$

48. Which of the following function satisfies Cauchy-Reimann equations.

    (a) $f(x, y) = (x^2 + y^2, 2xy)$
    (b) $f(x, y) = (sinxcosy, cosxsiny)$
    (c) $f(x, y) = (e^x cosy, e^x siny)$
    (d) $f(x, y) = (xyz, \frac{xy^2z}{2})$

49. The partial derivatives D_1f and D_2f of the function $f(x, y) = y^2e^x + x^2y^3 + 16$ at $(1, 2)$ is....

    (a) $4e + 16, 4e + 12$
    (b) $4e + 8, 4e + 16$
    (c) $2e^2 + 8, 2e^2 + 16$
    (d) $4e^2, 8e^2$

50. Gradient vector of the function $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ at $(1, 1)$ is ....

    (a) $(\frac{1}{2}, \frac{1}{2})$
    (b) $(2, 2)$
    (c) $(1, 1)$
    (d) $(\frac{1}{2}, \frac{-1}{2})$

51. The direction of $f(x, y) = x^2 + xy$ at $P_0(1, 2))$ in the direction of the unit vector $u = \frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j$ is ....

    (a) $\sqrt{2}$
    (b) $\frac{3}{\sqrt{2}}$
    (c) $\frac{5}{\sqrt{2}}$
    (d) 2

52. If $f : R^n \to R^m$ is a real valued function, then the Jacobian matrix consists of

    (a) only one column
    (b) only one row
    (c) both (a) and (b)

(d) none of the above.

53. The k^{th} row of the Jacobian matrix is a vector in R^n called ....

(a) the gradient vector of f_k

(b) constant vector of f_k

(c) unit vector of f_k

(d) none of the above

54. If $f(x) = \|x^2\|$ and $F(t) = f(c + tu)$, then $F^{'}(0) = ...$

(a) 0

(b) 2c

(c) $2c.u$

(d) none of the above

55. If g is differentiable at a with total derivative $g^{'}(a)$ and f is differentiable at $b = g(a)$ with total derivative $f^{'}(b)$, then $h = f \circ g$ is differentiable at a with the total derivative $h^{'}(a) =$....

(a) $f(b) \circ g^{'}(a)$.

(b) $f^{'}(b) \circ g(a)$.

(c) $g^{'}(b) \circ f^{'}(a)$.

(d) $f^{'}(b) \circ g^{'}(a)$.

56. If g is differentiable at a and f is differentiable at $b = g(a)$, then the matrix form of the chain rule for $h = f \circ g$ is ...

(a) $D(h(a)) = D(g(a)D(f(b))$.

(b) $D(h(a)) = D(f(a)D(g(b))$.

(c) $D(h(a)) = D(f(b)D(g(a))$.

(d) $D(h(a)) = D(g(b)D(f(a))$.

57. If $x, y \in R^n$, then $L(x, y)$ denotes .....

(a) $\{tx + (1 - t)y : 0 \leq t \leq 1\}$

(b) $\{tx + (1 - t)y : 0 < t < 1\}$

(c) $\{x + (1 - t)y : 0 \leq t \leq 1\}$

(d) $\{tx + (1 - t)y : 0 < t < 1\}$

58. By Mean-value theorem, if S be an open subset of R^n and $f : S \to R^m$ is differentiable at each point of S then :

(a) there exists a vector a in R^m and a point z in $L(x, y)$ such that $a \cdot \{f(y) - f(x)\} = a \cdot \{f'(z)(y - x)\}$, for any two points x and y in S.

(b) for every vector a in R^m and a point z in $L(x, y)$ such that $a \cdot \{f(y) - f(x)\} = a \cdot \{f'(z)(y - x)\}$, for any two points x and y in S such that $L(x, y) \subset S$.

(c) there is a point z in $L(x, y)$ such that $\{f(y) - f(x)\} = f'(z)$, for any two points x and y in S.

(d) there exist two points x and y in S and a point z in $L(x, y)$ such that$\{f(y) - f(x)\} = f^{'}(z)(y - x)$.

59. If S be an open convex subset of R^n and $f : S \to R^m$ is differentiable at each point of S then :

(a) there exists a vector a in R^m and a point z in $L(x, y)$ such that $a \cdot \{f(y) - f(x)\} = a \cdot \{f'(z)(y - x)\}$, for any two points x and y in S.

(b) for every vector a in R^m and a point z in $L(x, y)$ such that $a \cdot \{f(y) - f(x)\} = a \cdot \{f'(z)(y - x)\}$, for any two points x and y in S such that $L(x, y) \subset S$.

(c) there is a point z in $L(x, y)$ such that $\{f(y) - f(x)\} = f'(z)$, for any two points x and y in S.

(d) there exist two points x and y in S and a point z in $L(x, y)$ such that $\{f(y) - f(x)\} = \left\{f^{'}(z)(y - x)\right\}$.

60. Let S be and open connected subset of R^n, and let $f : S \to R^m$ be differentiable at each point of S then the necessary condition for f to be a constant on S is

(a) $f'(c) = 0$ for each c in S.

(b) $f'(c) \neq 0$ for each c in S.

(c) f is continuously differentiable.

(d) first derivative $f^{'}$ is continuous

61. Let S be and open connected subset of R^n, and let $f : S \to R^m$ be differentiable at each point of S with $f'(c) = 0$ for each c in S then

a) **f** is a constant on S

b) for every vector **a** there exists an z such that $\mathbf{a} \cdot \mathbf{f}'(z) = \mathbf{a}$

c) **f** is continuously differentiable on S.

d) first derivative $f^{'}$ is continuous

Module 3

62. A sufficient condition for $D_{r,k}f(c) = D_{k,r}f(c)$ is

a) one of the partial derivatives $D_r f$ and $D_k f$ exists in an n- ball $B(c;\delta)$ and are differentiable at c

b) both the partial derivatives $D_r f$ and $D_k f$ exists in an n- ball $B(c;\delta)$ and if both are continuous at c

c) both the partial derivatives $D_r f$ and $D_k f$ exists in an n- ball $B(c;\delta)$ and if both are differentiable at c

d) None of the above

63. A sufficient condition for $D_{r,k}f(c) = D_{k,r}f(c)$ is

a) one of the partial derivatives $D_r f$ and $D_k f$ exists in an n- ball $B(c;\delta)$ and are differentiable at c

b) both the partial derivatives $D_r f$ and $D_k f$ exists in an n- ball $B(c)$ and if both $D_{r,k}f(c)$ and $D_{k,r}f(c)$ are continuous at c

c) both the partial derivatives $D_r f$ and $D_k f$ exists in an n- ball $B(c;\delta)$ and if both are differentiable at c

d) Both (b) and (c) are sufficient conditions

64. If $f = u + iv$ is a complex valued function with a derivative at a point z in C, then

a) $J_f(z) = |f'(z)|^2$.

b) $J_f(z) = |f^2(z)|$.

c) $J_f(z) = |f'(z)^2|$.

d) $J_f(z) = |f(z)|^2$.

65. If $f(x, y) = (xe^y, xy)$ then the Jacobian determinant is

a) xye^y

b) xe^y

c) xy^2e^y

d) $x(1 - y)e^y$

66. If $f = u + iv$ then the Jacobian determinant is

a) $D_1uD_1v - D_2vD_2u$

b) $D_1uD_1v - D_2uD_2v$

c) $D_1uD_2v - D_1vD_2u$

d) $D_2uD_2v - D_1uD_1v$

67. Let $f : S \to R$ be a real valued function. Assume that f is continuous on a compact subset X of S. Then there exist points p and q in X such that

a) $f(p) = \sup f(X)$ and $f(q) = \sup f(X)$.

b) $f(p) = \inf f(X)$ and $f(q) = \sup f(X)$.

c) $f(p) = \text{Max} f(X)$ and $f(q) = \text{Min} f(X)$.

d) $f(p) = f(X)$ and $f(q) = 0$.

68. The bountary ∂B of an n-ball $B = B(a; r)$ is given by

a) $\partial B = \{X : |x - a| = r\}$

b) $\partial B = \{X : \|x - a\| = r\}$

c) $\partial B = \{X : |x - a| < r\}$

d) $\partial B = \{X : \|x - a\| < r\}$

69. A function $f : S \to T$ is called an open mapping if,

a) for every open set A in S, the image $f(A)$ is open in T.

b) for every set A in S, the image $f(A)$ is open in T.

c) for every open set A in T, the inverse image $f^{-1}(A)$ is open in S.

d) for every set A in T, the inverse image $f^{-1}(A)$ is open in S.

70. Let A be an open subset of R^n and assume that $f : A \to R^n$ is continuous and has finite partial derivatives $D_j f_i$ on A then the sufficient condition for $f(A)$ to be open is

a) f is continuous on A and if $J_f(x) \neq 0$ for each $x \in A$

b) f is onto on A and if $J_f(x) \neq 0$ for each $x \in A$

c) f is one to one on A and if $J_f(x) \neq 0$ for each $x \in A$

d) none of the above

71. Let A be an open subset of R^n and assume that $f : A \to R^n$ is continuous and has finite partial derivatives $D_j f_i$ on A. If f is one to one on A and if $J_f(x) \neq 0$ for each $x \in A$, then

a) $f(A)$ is closed

b) $f(A)$ is open

c) $f(A)$ is both open and closed

d) All of the above

72. Assume that $f = (f_1, f_2, \ldots, f_n)$ has continuous partial derivatives $D_j f_i$ on an open set S in R^n, and that the Jacobian determinant $J_f(a) \neq 0$ for some point a in S then

a) there is an n-ball $B(a)$ on which f is one to one.

b) f is one to one on S.

c) f is onto

d) None of the above

73. Let A be an open subset of R^n and assume that $f : A \to R^n$ has continuous partial derivatives $D_j f_i$ on A and if $J_f(x) \neq 0$ for all x in A, then

a) $f : A \to R^n$ has continuous partial derivatives $D_j f_i$ on A

b) $J_f(x) \neq 0$ for all x in A

c) f is an open mapping

d) All of the above

74. Let A be an open subset of R^n and assume that $f : A \to R^n$ has continuous partial derivatives $D_j f_i$ on A then the sufficient condition for f to be an open mapping is

a) $f : A \to R^n$ has continuous partial derivatives $D_j f_i$ on A

b) $J_f(x) \neq 0$ for all x

c) both (a) and (b)

d) none of the above

75. If a function $f = (f_1, \ldots, f_n)$ has continuous partial derivatives on a set S, then we say that

a) f is continuously differentiable on S

b) f is differentiable on S

c) f has total derivative on S

d) None of the above

76. Assume that the second order partial derivatives $D_{i,j}$ exist in an n-ball $B(a)$ and are continuous at a stationary point a of f and let $Q(t) = \frac{1}{2} f''(a;t)$. Then f has a relative minimum at a if ....

(a) $Q(t) < 0$ for all $t \neq 0$.

(b) $Q(t) > 0$ for all $t \neq 0$.

(c) $Q(t) = 0$ for all $t \neq 0$.

(d) $Q(t)$ takes both positive and negative values.

77. Assume that the second order partial derivatives $D_{i,j}$ exist in an n-ball $B(a)$ and are continuous at a stationary point a of f and let $Q(t) = \frac{1}{2} f''(a;t)$. Then f has a relative maximum at a if ....

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(d) $Q(t)$ takes both positive and negative values.

78. Assume that the second order partial derivatives $D_{i,j}$ exist in an n-ball $B(a)$ and are continuous at a stationary point a of f and let $Q(t) = \frac{1}{2}f''(a;t)$. Then f has a saddle point at a if ....

(a) $Q(t) < 0$ for all $t \neq 0$.

(b) $Q(t) > 0$ for all $t \neq 0$.

(c) $Q(t) = 0$ for all $t \neq 0$.

(d) $Q(t)$ takes both positive and negative values.

79. Let f be a real valued function with continuous second order partial derivatives at a stationary point a in R^2 and let $\Delta = AC - B^2$, where $A = D_{1,1}f(a), B = D_{1,2}f(a), C = D_{2,2}f(a)$. Then f has a relative minimum at a if ....

(a) if $\Delta > 0$ and $A < 0$.

(b) if $\Delta > 0$ and $A > 0$.

(c) if $\Delta < 0$ and $A < 0$.

(d) if $\Delta < 0$ and $A > 0$.

80. Let f be a real valued function with continuous second order partial derivatives at a stationary point a in R^2 and let $\Delta = AC - B^2$, where $A = D_{1,1}f(a), B = D_{1,2}f(a), C = D_{2,2}f(a)$. Then f has a relative maximum at a if ....

(a) if $\Delta > 0$ and $A < 0$.

(b) if $\Delta > 0$ and $A > 0$.

(c) if $\Delta < 0$ and $A < 0$.

(d) if $\Delta < 0$ and $A > 0$.

81. Let f be a real valued function with continuous second order partial derivatives at a stationary point a in R^2 and let $\Delta = AC - B^2$, where $A = D_{1,1}f(a), B = D_{1,2}f(a), C = D_{2,2}f(a)$. Then f has a saddle point at a if ....

(a) if $\Delta > 0$ and $A < 0$.

(b) if $\Delta > 0$ and $A > 0$.

(c) if $\Delta < 0$.

(d) if $\Delta > 0$.

82. A quadratic form $\Sigma_{i=1}^{n}\Sigma_{j=1}^{n}a_{ij}x_ix_j$ is negative definite if

(a) $x \neq 0 \Rightarrow Q(x) \leq 0.$

(b) $x < 0 \Rightarrow Q(x) < 0.$

(c) $x \neq 0 \Rightarrow Q(x) < 0.$

(d) $x \neq 0 \Rightarrow Q(x) \leq 0.$

83. A quadratic form $\Sigma_{i=1}^{n}\Sigma_{j=1}^{n}a_{ij}x_ix_j$ is positive definite if

(a) $x \neq 0 \Rightarrow Q(x) \geq 0.$

(b) $x > 0 \Rightarrow Q(x) > 0.$

(c) $x \neq 0 \Rightarrow Q(x) \geq 0.$

(d) $x \neq 0 \Rightarrow Q(x) > 0.$

84. Which of the following is not a quadratic form in R^n?

(a) $x_1^2 + x_2^2 + ... + x_n^2$

(b) $\Sigma_{i=1}^{3}\Sigma_{j=1}^{3}a_{ij}x_ix_j$, where a_{ij} are constants.

(c) $x_1x_2 + x_2x_3 + ... + x_nx_{n+1}$

(d) None of the above

Module 4

85. A linear operator B on R^n that interchanges some pair of members of the standard basis and leaves the others fixed is called.....

(a) k-cell

(b) Flip

(c) Primitive Map

(d) None of the above

86. Choose the correct statement from the following:

(a) A k- cell in R^k is defined as the set of all points $\mathbf{x} = (x_1, x_2, \ldots, x_k)$ such that $a_i \leq x \leq b_i, i = 1, 2, 3, \ldots, k.$

(b) A rectangle is a 2-cell.

(c) A k-cell is the product of intervals of the form $[a_i, b_i], i = 1, 2, \ldots, k.$

(d) All of the above.

87. For every $f \in C(I^K), L(f) = ...$

(a) $L^{'}(f^{'})$

(b) $L(f^{'})$

(c) $L^{'}(f)$

(d) none of the above

88. Let Q^k be the k-simplex which consists of all points $\mathbf{x} = (x_1, ..., x_k)$ in R^k for which $x_1 + ... + x_k \leq 1$ and $x_i \geq 0$. Then Q^3 is a ....

(a) triangle

(b) line Segment

(c) tetrahedron

(d) a point

89. Pick the correct statement(s)

(a) For every $f \in C(I^k)$, the order of integration is immaterial for $\int_{I_k} f(\mathbf{x})d\mathbf{x}$.

(b) If f is a continuous function with a compact support, then the integral $\int_{R^k} = \int_{I_k} f$ is independent of the choice of I^k provided that I^k contains the support of f.

(c) Both (a) and (b)

(d) None of the above.

90. The closure of the set of all points $\mathbf{x} \in R^k$ at which $f(\mathbf{x}) \neq 0$ is.....

(a) k-cell

(b) unit cell

(c) support of f

(d) flip

91. The support of a real or complex function f on R^k is ....

(a) The closure of the set of all points $\mathbf{x} \in R^k$ at which $f(\mathbf{x}) \neq 0$.

(b) The closure of the set of all points $\mathbf{x} \in R^k$ at which $f(\mathbf{x}) = 0$.

(c) The set of all points $\mathbf{x} \in R^k$ at which $f(\mathbf{x}) \neq 0$.

(d) The of all points $\mathbf{x} \in R^k$ at which $f(\mathbf{x}) = 0$.

92. Let G be a primitive mapping of the form $G(\mathbf{x}) = \mathbf{x} + [g(\mathbf{x}) - x_m]\, e_m$, where g is a real function with domain E. Then $G^{'}(a)$ is invertible if

(a) $(D_m g)(a) = 0$

(b) $(D_m g)(a) = 1$

(c) $(D_m g)(a) \neq 0$

(d) none of the above

93. Let G be a primitive mapping of the form $G(\mathbf{x}) = \mathbf{x} + [g(\mathbf{x}) - x_m]\, e_m$, where g is a real function with domain E. Then

(a) $J_G(a) = (D_m g)(a)$.

(b) $(D_m g)(a) = I$

(c) $(D_m g)(a) \neq 0$

(d) none of the above

94. Integrals of 1- forms are called ....

(a) Integrals on I^k

(b) Integrals over R^k

(c) Line Integrals

(d) Surface Integrals.

95. The support of a function f on R^k is the closure of the set of all points $\mathbf{x} \in R^k$ at which.....

(a) $f(\mathbf{x}) \neq 0$

(b) $f(\mathbf{x}) = 0$

(c) $f(\mathbf{x}) = 1$

(d) none of the above

96. Choose the correct statement from the following statements:

(a) Suppose E is an open set in R^n. A k-surface in E is a $\mathcal{C}'$ mapping ϕ from a compact set $D \subset R^k$ into E. D is called the parameter domain of ϕ.

(b) A primitive mapping is the one that changes at most one coordinate.

(c) Both (a) and (b)

(d) None of these.

97. A k-surface in $E \subset R^k$ is ......

(a) a mapping ϕ from a compact set $D \subset R^k$ into E.

(b) a $\mathcal{C}'$ mapping ϕ from a compact set $D \subset R^k$ into E.

(c) a $\mathcal{C}'$ mapping ϕ from a set $D \subset R^k$ into E.

(d) a mapping ϕ from a compact set $D \subset R^k$ into E.

98. Let D be the 3-cell defined by $0 \leq r \leq 1, 0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi$ and define $\Phi(r, \theta, \phi) = (x, y, z)$, where $x = rsin\theta cos\phi, y = rsin\theta sin\phi, z = rcos\theta$. Then $J_\Phi(r, \theta, \phi) = ...$

(a) $rsin\theta$

(b) $r^2 sin\theta$

(c) $2rsin\theta$

(d) $r^2 cos\theta$

99. Let D be the 3-cell defined by $0 \leq r \leq 1, 0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi$ and define $\Phi(r, \theta, \phi) = (x, y, z)$, where $x = rsin\theta cos\phi, y = rsin\theta sin\phi, z = rcos\theta$. Then $\int_\Phi dx \wedge dy \wedge dz = ...$

(a) $\frac{4\pi}{3}$

(b) $\frac{\pi}{3}$

(c) 4π

(d) $\frac{3\pi}{4}$

100. Let ω_1, ω_2 be k-forms in E. Then $\omega_1 = \omega_2$ if and only if.....

(a) $\omega_1(\phi) = \omega_2(\phi)$ for every k-surface ϕ in E.

(b) $\omega_1(\phi) = \omega_2(\phi)$ for some k-surface ϕ in E.

(c) $\omega_1(\phi) = \omega_2(\phi)$ for atleast one k-surface ϕ in E.

(d) none of the above

101. A basic k-form in R^n is of the form $dx_I = dx_{i_1} \wedge ... \wedge dx_{i_k}$, where I is the ordered k-tuple such that

(a) $1 \leq i_1 \leq i_2 \leq ... \leq i_k \leq n.$

(b) $1 \leq i_1 < i_2 < ... < i_k \leq n.$

(c) $1 \geq i_1 > i_2 > ... > i_k \geq n.$

(d) $1 \geq i_1 \geq i_2 \geq ... \geq i_k \leq n.$

102. Which of the following is a basic k-form ?

(a) $dx_1 \wedge dx_5 \wedge dx_3$

(b) $dx_2 \wedge dx_3 \wedge dx_4$

(c) $dx_4 \wedge dx_3 \wedge dx_2$

(d) $dx_3 \wedge dx_2 \wedge dx_1$

103. The standard presentation of the 2-form $w = x_1 dx_2 \wedge dx_1 - x_2 dx_3 \wedge dx_2 + x_3 dx_2 \wedge dx_3 + dx_1 \wedge dx_2$ is

(a) $(x_1 - 1)dx_1 \wedge dx_2 + (x_2 + x_3)dx_2 \wedge dx_3$

(b) $(x_1 - 1)dx_2 \wedge dx_1 + (x_2 + x_3)dx_3 \wedge dx_2$

(c) $(1 - x_1)dx_1 \wedge dx_2 + (x_2 + x_3)dx_2 \wedge dx_3$

(d) $(1 - x_1)dx_2 \wedge dx_1 + (x_2 + x_3)dx_3 \wedge dx_2$

104. Let ω be a k-form in E. Then for every k-surface Φ in E, we have

(a) $\int_\phi \omega = -\int_\phi \omega$

(b) $\int_\phi \omega = -\int_\phi \omega + c$ for some positive constant c.

(c) $\int_\phi (-\omega) = -\int_\phi \omega$

(d) $\int_\phi \omega = -\int_\phi \omega - c$ for some positive constant c.

105. Whch of the following is true for differential forms in R^n.

(a) $dx_i \wedge dx_i \neq 0$

(b) $dx_i \wedge dx_i = 1$

(c) $dx_i \wedge dx_j = dx_j \wedge dx_i$

(d) $dx_i \wedge dx_j = -dx_j \wedge dx_i$

106. Consider the k-form $w = a(x)dx_{i_1} \wedge ... \wedge dx_{i_k}$. Then w=0 if ....

(a) $i_1 \neq i_k$.

(b) The subscripts $i_1, i_2, ..., i_k$ are all distinct.

(c) Any two suffices among $i_1, i_2, ..., i_k$ are equal.

(d) None of the above.

107. Whch of the following is wrong?

(a) $dx_i \wedge dx_j = -dx_j \wedge dx_i$

(b) If $k > n$, then then the only k-form in any open subset of R^n is 0.

(c) $dx_{i_1} \wedge ... \wedge dx_{i_k} = 0$ unless the subscripts $i_1, ..., i_k$ are all distinct.

(d) None of the above.