## ME010301 - Advanced Complex Analysis

- 1. If *u* is a harmonic function in a region *D*,  $\partial D$  denotes the boundary of *D* and  $\overline{D} = D \cup \partial D$ , then
  - A)  $\max_{\overline{D}} u \ge \max_{D} u$
  - B)  $\max_{\overline{D}} u = \max_{\partial D} u$
  - C)  $\max_{D} u = u(x) \forall x \in D$
  - D) u is a constant in D
- 2. The value of *b* for which  $u(x, y) = e^{bx} \cos 5y$  is harmonic
  - A) 5
  - B) 6
  - Ć) 7
  - D) 8
- 3. Solutions of Laplace's equation having continuous second order partial derivatives are called
  - A) Biharmonic functions
  - B) Harmonic functions
  - C) Conjugate harmonic functions
  - D) Error functions
- 4. What is the value of m for which  $2x x^2 + my^2$  is harmonic
  - A) 1
  - B) -1
  - C) 2
  - D) -2
- 5. If  $u(x,y) = 2x^2 2y^2 + 4xy$  is a harmonic function then its conjugate harmonic function is
  - A)  $4xy 2x^2 + 2y^2 + C$
  - B)  $4y^2 4xy + C$
  - C)  $2x^2 2y^2 + xy + C$
  - D)  $-4xy 2x^2 + 2y^2 + C$
- 6. If  $u(x, y) = x^3 + ax^2y + bxy^2 + 2y^3$  is a harmonic function then its conjugate harmonic function is
  - A)  $4xy 2x^2 + 2y^2 + C$
  - B)  $4y^2 4xy + C$
  - C)  $2x^2 2y^2 + xy + C$
  - D)  $3x^2y 6xy^2 y^3 + 2x^3 + C$
- 7. If  $u(x, y) = x^2 y^2$  is a harmonic function then its conjugate harmonic function is A)  $-x^2 + y^2$ 
  - B)  $x^2 + y^2$
  - C) 2xy
  - D)  $-x^2 y^2$
- 8. A function u is said to be harmonic if
  - A)  $u_{xx} + u_{yy} = 0$
  - B)  $u_{xy} + u_{yx} = 0$
  - $C) \ u_x + u_y = 0$
  - D)  $u_x^2 + u_y^2 = 0$
- 9. If u and v are harmonic functions then f(z) = u + iv is
  - A) Analytic function
  - B) Need not be Analytic function
  - C) Analytic function only at z = 0
  - D) None of the above

- 10. A function v is called a conjugate harmonic function for a harmonic function u in  $\Omega$  whenever
  - A) f = u + iv is analytic
  - B) *u* is analytic
  - C) v is analytic
  - D) None of the above
- 11. Assume that u is harmonic in a region  $\Omega$  and for  $z_0 \in \Omega$ ,  $|z z_0| < r \subset \Omega$  then the average value of u over the boundary of the disc is
  - A)  $u(z_0)$
  - B)  $ru(z_0)$
  - C)  $\frac{u(z)}{\pi r}$

D) 
$$\frac{4}{3}\pi r^{3}u(z_{0})$$

- 12. The function f(z) of complex variable z is given  $asf(z) = x^3 3xy^2 + iv(x, y)$ . For this function to be analytic v(x, y) should be
  - A)  $3xy^2 y^3 + C$
  - B)  $3x^2y^2 y^3 + C$
  - C)  $x^3 3xy^2 + C$
  - D)  $3x^2y y^3 + C$
- 13. Which of the following functions u + iv are not analytic, given that u and v are harmonic?
  - A)  $u = \log(x^2 + y^2)$ ,  $v = 2tan^{-1}\left(\frac{y}{x}\right)$
  - B)  $u = 2xy, v = x^2 y^2$
  - C)  $u = e^y cosx$ ,  $v = -e^y sinx$
  - D)  $u = 2x(1-y), v = x^2 y^2 + 2y$
- 14. Which of the following functions u + iv are analytic, given that u and v are harmonic?
  - A)  $u = x^2 y^2$ ,  $v = \frac{-y}{x^2 + y^2}$

B) 
$$u = e^y \cos x, v = -e^x \cos y$$

C) 
$$u = log\sqrt{x^2 + y^2}, v = tan^{-1}\left(\frac{y}{x}\right)$$

D)  $u = 2xy, v = x^2 - y^2$ 

15. Which of the following is/are true for analytic function f(z) = u + iv

- A) u is harmonic function
- B) v is harmonic function
- C) v is the conjugate harmonic function of u
- D) all of above
- 16. A continuous function u(z) which satisfies the mean value property is
  - A) Analytic
  - B) Entire
  - C) Harmonic
  - D) Subharmonic
- 17. If u(z) is a harmonic function and v(z) is a subharmonic function defined in a region  $\Omega$ , then
  - A) u(z) = v(z)
  - B)  $u(z) \le v(z)$
  - C)  $u(z) \ge v(z)$
  - D) None of the above
- 18. v(z) is a subharmonic function if
  - A)  $\Delta v > 0$
  - B)  $\Delta v < 0$
  - C)  $\Delta v = 0$
  - D) None of the above
- 19. A continuous function v(z) is subharmonic in  $\Omega$  if and only if

A) 
$$v(z_0) \ge \frac{1}{2\pi} \int_0^{2\pi} v(z_0 + re^{i\theta}) d\theta$$

- B)  $v(z_0) = \frac{1}{2\pi} \int_0^{2\pi} v(z_0 + re^{i\theta}) d\theta$ C)  $v(z_0) \le \frac{1}{2\pi} \int_0^{2\pi} v(z_0 + re^{i\theta}) d\theta$
- D) None of the above
- 20. Which of the following statements are not true if  $v_1$  and  $v_2$  are two subharmonic functions
  - A)  $kv_1$  is subharmonic
  - B)  $v_1 + v_2$  is subharmonic
  - C)  $\max(v_1, v_2)$  is subharmonic
  - D) min( $v_1, v_2$ ) is subharmonic
- 21. A continuous function v(z) in a region  $\Omega$  satisfies the maximum principle means
  - A) v attains its maximum at a point in  $\Omega$
  - B) v attains its minimum at a point in  $\Omega$
  - C) v can have no maximum at a point in  $\Omega$
  - D) None of the above
- 22. In polar coordinates the Laplace's equation is

A) 
$$r \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial \theta^2} = 0$$
  
B)  $r \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{\partial u}{\partial \theta} = 0$   
C)  $r \frac{\partial}{\partial r} \left( r \frac{\partial^2 u}{\partial r^2} \right) + \frac{\partial^2 u}{\partial \theta^2} = 0$   
D) None of the above

- 23. If  $u \ge 0$  is any continuous function then the Poisson integral  $P_u(z)$  satisfies
  - A)  $P_u(z) = 0$
  - B)  $P_u(z) \leq 0$
  - C)  $P_u(z) \geq 0$
  - D) None of the above
- 24. If u is any continuous function which satisfies  $m \le u \le M$ , then the Poisson integral  $P_{\mu}(z)$  satisfies
  - A)  $m \ge P_u(z) \ge M$
  - B)  $m \le P_u(z) \le M$
  - C)  $m = P_u(z) = M$
  - D) None of the above
- 25. The arithmetic mean of a harmonic function over concentric circles |z| = r is
  - A)  $\alpha logr + \beta$
  - B)  $\sqrt{\alpha logr}$
  - C)  $e^{r\beta}$
  - D) None of the above
- 26. Limit of a uniformly convergent sequence of analytic function is
  - A) Analytic
  - B) Differentiable
  - C) Continuous
  - D) Not analytic

27. Laurent series expansion of  $f(z) = \frac{1}{(z-1)(z-2)}$  valid in the region 1 < |z| < 2 is

 $\frac{-1}{z} \left( 1 + \frac{z}{2} + \frac{z^2}{4} + \cdots \right) + \frac{1}{2} \left( 1 + \frac{1}{z} + \frac{1}{z^2} + \cdots \right)$ A)  $\frac{-1}{2}\left(1 + \frac{z}{2} + \frac{z^2}{4} + \cdots\right) + \frac{1}{z}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \cdots\right)$ B) C)  $\frac{\frac{-1}{2}}{2}\left(1+\frac{z}{2}+\frac{z^{2}}{4}+\cdots\right) - \frac{1}{z}\left(1+\frac{1}{z}+\frac{1}{z^{2}}+\cdots\right)$  $\frac{\frac{1}{z}}{z}\left(1 + \frac{z}{2} + \frac{z^2}{4} + \cdots\right) - \frac{1}{2}\left(1 + \frac{1}{z} + \frac{1}{z^2} + \cdots\right)$ D)

28. Laurent series of function  $f(z) = \frac{1}{z(z-1)}$  for region |z| < 1 is

- $1 z z^2 \dots$ A)
- B)  $\frac{1}{z} + 1 + z + z^2 + ...$ C)  $-\frac{1}{z} 1 z z^2 ...$

29. Principal part of Laurent series of  $f(z) = \frac{e^z - 1}{z}$  is

- A) Z 1 B)
- $_{1}^{z}$
- C)
- D) 0

30. Mittag-Lefflers theorem states

- Construction of meromorphic function A)
- f is an irrational function B)
- C) f is a rational function
- D) f is a trigonometric function

31. Poles of  $f(z) = \frac{\pi^2}{\sin^2 \pi z}$  are given by

- A) z=n
- B)  $z=n\pi$
- $Z = \frac{1}{n}$  $Z = \frac{-1}{n}$ C)
- D)

32. Taylor series expansion of f(x+h) in powers of h is A)  $f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \dots$ B)  $f(x) + h f'(x) + h2 f''(x) + \dots$ C)  $f(x) + h f'(x) - \frac{h^2}{2!} f''(x) + ...$ D)  $f(x) - h f''(x) + \frac{h^2}{2!} f''(x) - ...$ 

33. Taylor series expansion of f(x) in powers of (x-a) is

A) 
$$f(a) - (x - a)f'(a) + \frac{(x - a)^2}{2!}f''(a) - \frac{(x - a)^3}{3!}f'''(a) + \dots$$
  
B)  $f(a) + (x - a)f'(a) - \frac{(x - a)^2}{2!}f''(a) + \frac{(x - a)^3}{3!}f'''(a) + \dots$ 

B) 
$$f(a) + (x - a) f'(a) - \frac{(x - a)}{2!} f''(a) + \frac{(x - a)}{3!} f'''(a) + \dots$$
  
C)  $f(a) + (x - a) f'(a) + \frac{(x - a)^2}{2!} f''(a) + \frac{(x - a)^3}{3!} f'''(a) + \dots$ 

C) 
$$f(a) + (x - a) f'(a) + \frac{(x - a)}{2!} f''(a) + \frac{(x - a)^2}{3!} f'''(a) + ...$$
  
D) None of the above

D) None of the above

34. Taylor series expansion of  $f(x) = e^x$  is given by A)  $1 + x + x^2 + x^3 + x^3$ 

A) 
$$1 + x + x^2 + x^3 + ...$$
  
B)  $1 - x + x^2 - x^3 + ...$   
C)  $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$   
D)  $1 - x - \frac{x^2}{2!} - \frac{x^3}{3!} + ...$ 

D)  $1 - x - \frac{x^2}{2!} - \frac{x^2}{3!} + \dots$ 35. Taylor series expansion of  $f(x) = \sin x$  is

A) 
$$1 + \frac{X^2}{2!} + \frac{X^3}{3!} + \dots$$
  
B)  $1 - \frac{X^2}{2!} + \frac{X^4}{4!} + \dots$   
C)  $x - \frac{X^3}{3!} + \frac{x^5}{5!} - \dots$   
D)  $x + \frac{X^3}{3!} + \frac{x^5}{5!} + \dots$ 

36. The Maclaurin's series expansion of  $f(x) = e^{sinx}$  is

A)  $1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \dots$ B)  $1 + \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^4}{8} + \dots$ C)  $1 + x + x^2 + x^3 + \dots$ D)  $1 - x + -\frac{x^2}{2} + \frac{x^3}{3} + \dots$ 

D) 
$$1 - x + -\frac{x^2}{x} + \frac{x}{x}$$

37. Gamma function have

- Only one zero. A)
- More than one zeros but finitely many. B)

C) No zeros

D) Infinitely many zeros.

38. The poles of  $\Gamma$  (*z*) are

A) z = 0, -1, -2, -3, ...

- B) z = 0, 1, 2, 3, ...
- C) z = 1, 2, 3, 4, ...
- D)  $z = 1, \frac{1}{2}, \frac{1}{3}, ...$

39. Every entire function which is meromorphic in the whole plane is

- A) The product of two entire functions
- B) Quotient of two entire functions
- C) Sum of two entire functions
- D) Difference of two entire functions

40. Number of zeros of gamma function is

- A) 1
- B) ∞
- C) 0
- D) finite

41.  $sin \pi z$  is an entire function of genus

- A) 1
- B) 2
- C) 0

42. Genus of an entire function of the form  $C z^m \prod_{n=1}^{\infty} (1 - \frac{z}{an})$  with  $\sum_{n=1}^{\infty} \frac{1}{|an|} < \infty$ 

- A) 0 B) 1
- B)
- C) D)

D) 3 43. Find the value of  $\lim_{m \to \infty} \sum_{n=-m}^{m} \frac{(-1)^n}{z-n}$ 

A)  $\frac{\pi}{\sin \pi z}$ B)  $\cot \pi z$ 

2

- C  $\pi$
- C)  $\frac{\pi}{\cos \pi z}$
- D) None of the above
- 44. Value of  $\Gamma(n)$  is
  - A) (n-1)!
  - B) n!
  - C) (n+1)!
  - D) (2n-1)!

45. The value of  $\Gamma(\frac{1}{2})$  is

- A)  $\sqrt{\pi}$
- B)  $\sqrt{2\pi}$
- C)  $\sqrt{4\pi}$
- D)  $\sqrt{\frac{\pi}{2}}$

46. Find the value of  $e^{\pi z \cot \pi \alpha} \prod_{-\infty}^{\infty} (1 + \frac{z}{n+\alpha}) e^{-\frac{z}{n+\alpha}}$ 

- A)  $\sin \pi z$
- B)  $\sin \pi (z + \alpha)$
- C)  $\cos \pi z$
- D)  $\cos \pi(z + \alpha)$

47. In Taylor series expansion of  $e^x$  about x = 2, the coefficient of  $(x-2)^4$  is

A)  $\frac{2^4}{4!}$ B)  $\frac{1}{4!}$ C)  $\frac{e^2}{4!}$   $e^4$  4!

D)

48. The expansion of arc sin z is

- $Z + \frac{1}{2}\frac{z^{3}}{3} + \frac{1.3}{2.4}\frac{z^{5}}{5} + \frac{1}{2.4.6}\frac{3.5}{7}z^{7} + \dots$  $Z \frac{z^{3}}{3} \frac{1.3}{2.4}\frac{z^{5}}{5} + \frac{1.3.5}{2.4.6}\frac{z^{7}}{7} \dots$  $Z + \frac{1.3}{2.4}\frac{z^{5}}{5} + \frac{1.3.5}{2.4.6}\frac{z^{7}}{7} + \dots$ A) B)
- C)
- D) None of the above
- 49. Find the expansion of  $\frac{1}{\sqrt{1-z^2}}$ A)  $1 + \frac{1}{2}z^2 + \frac{1.3}{2.4}z^4 + \frac{1.3.5}{2.4.6}z^6 + \dots$ B)  $1 \frac{1}{2}z^2 \frac{1.3}{2.4}z^4 \frac{1.3.5}{2.4.6}z^6 \dots$ C)  $1 + z^2 + \frac{1.3}{2.4}z^4 + \frac{1.3.5}{2.4.6}z^6 + \dots$ 

  - None of the above D)
- 50. Find the value of  $1 \frac{1}{3} + \frac{1}{5} \frac{1}{7} + \cdots$ 
  - A)
  - B) π π
  - C)
  - D)

5 51. State in which region the Riemann zeta function is analytic.

- A)  $\sigma > 1$
- B)  $\sigma < 1$
- C)  $\sigma = 1$
- D) None of this

3 2π

52. Closure of a family with respect to a distance function is compact then the family is .....

- A) normal
- B) totally bounded
- C) equicontinuous
- D) Bounded
- 53. The theorem says about the connection between equicontinuity and normality A) Legendre 's relation
  - B) Arzela's Theorem
  - C) Cauchy residue theorem
  - D) Riemann mapping theorem
- 54. Zeta (0) will take the value
  - A) -1/2
  - B) 0
  - C) -1/12
  - D) 2

55.

 $\zeta(s) = 2^s \pi^{s-1} \sin(\frac{\pi s}{2}) \Gamma(1-s) \zeta(1-s)$  which is known as\_\_\_\_\_

- A) Riemann functional equation
- B) Lebesgue integral functional equation
- C) zeta functional equation
- D) zero functional equation

56. The family F is normal if and only if F closure is\_

- A) zero
- B) constant
- C) compact
- D) normal
- 57. A family F is normal in  $\Omega$  if every sequence  $\{f_n\}$  of functions  $f_n \in F$  consists a subsequence which converge uniformly on\_
  - A) Boundary of  $\Omega$

- B) compact subset of  $\Omega$
- C) normal subset of  $\Omega$
- D) subset of  $\Omega$
- 58. Which theorem establishes the relationship between equicontinuity and normality of a family of functions?
  - A) Bolzano Weierstrass theorem
  - B) Arzela's theorem
  - C) Cauchy's theorem
  - D) Hadamard's theorem
- 59. The function z<sup>n</sup> where n is a nonnegative integer form a normal family in which region?
  - A) |z|<1

B) |z| >1

- C) both A and B
- D) none of the above
- 60. Is a zeta function can be extended to a meromorphic function? Give its pole and residue
  - A) Yes, 1,1
  - B) Yes 1,0
  - C) Yes 0,0
  - D) None of the above
- 61. Riemann Conjecture asserts that all nontrivial zeros lie in the critical line

$$\sigma = \frac{1}{2}$$

B) 
$$\sigma = 1$$

- C)  $\sigma < 1$
- D)  $\sigma = -2$
- 62. Zeta function has
  - A) Infinitely many zeros
  - B) Finitely many zeros
  - C) 0 as a zero
  - D) No real zeroes
- 63. Which of the following is true
  - A) Zeta function has zeros in the half plane S>1
  - B) Zeta function has no zeros in the half plane S<0
  - C) Zeta function has no zeros in the half plane S>1
  - D) Zeta function has no zeros in S>0
- 64. A family S of analytic functions is normal with respect to C if and only if
  - A) the functions in S are uniformly bounded on a set.
  - B) the functions in S are uniformly bounded on every compact set.
  - C) the functions in S are uniformly bounded on every closed set.
  - D) the functions in S are bounded on every set.
- 65. The family of functions F is normal if and only if its closure is \_\_\_\_\_
  - A) Constant
  - B) Compact
  - C) Normal
  - D) Relatively compact
- 66. A sequence  $\{f_n\}$  of functions  $f_n \in F$  converges with respect to a metric (distance function)  $\rho$  if and only if  $\{f_n\}$  on all compact subset of  $\Omega$ .
  - A) Converges uniformly
  - B) Diverges
  - C) Strictly increasing
  - D) Bounded
- 67. If a metric space S is complete, then the family of functions F with values in S is normal if and only if F is \_\_\_\_\_
  - A) Relatively compact
  - B) Totally bounded

C) Normal

D) Compact

68. Each function in an equicontinuous family is

A) absolutely continuous

B) not absolutely continuous

C) uniformly continuous

D) not uniformly continuous

## 69. Every finite set of uniformly continuous function is

A) equicontinuous

B) uniformly equicontinuous

C) not equicontinuous

D) not uniformly equicontinuous

70.  $\zeta(s) = 0$  when s is  $-2, -4, -6, \dots$  these numbers are called

A) trivial zeros

B) constant functions

C) zero functions

D) continues functions

71. F is normal iff F is relatively \_\_\_\_\_.

A) compact

B) constant

C) zero

- D) None of these
- 72. A sequence  $\{f_n\}$  converges with respect to metric  $\delta$  if and only if the sequence  $\{f_n\}.....$ 
  - A) diverges
  - B) converges
  - C) constant
  - D) None of these
- 73. A family F is\_\_\_\_\_ if for every  $\epsilon > 0$  there exists finite no of functions  $f_1, f_2, ..., f_n$  such that every  $f \in F$  satisfies  $\rho(f, f_i) < \epsilon$  for some  $f_j, j \in \{1, 2, ..., n\}$ 
  - A) totally bounded
  - B) equicontinuous
  - C) zero
  - D) normal
- 74. The  $\zeta$ -function can be extended to a meromorphic function in the whole plane whose only pole is a simple pole at s = 1 with the residue \_\_\_\_\_.
  - A) -1
  - B) 0
  - C) 1
  - D) ½
- 75. A family of continuous functions with values in a metric space S is normal in the region of the complex plane if and only if it is \_\_\_\_\_\_ on every compact set.
  - A) connected
  - B) equicontinuous
  - C) uniformly continuous
  - D) totally bounded
- 76. An analytic function g(z) in  $\Omega$  is said to be univalent if
  - A)  $g(z_1) = g(z_2)$  only for  $z_1 = z_2$
  - B) g is one to one
  - C) g is an injection
  - D) All of the above
- 77. Let f be a topological mapping of a region  $\Omega$  onto a region  $\Omega'$ . If  $\{z_n\}$  or z(t) tends to the boundary of  $\Omega$ , then  $\{f(z_n)\}$  or f(z(t))

- A) Diverges to infinity
- B) Tends to the boundary of  $\Omega'$
- C) Tends to an interior point of  $\Omega'$
- D) None of the above
- 78. Suppose that the boundary of a simply connected region  $\Omega$  contains a line segment  $\gamma$  as a one-sided free boundary arc. Then the function f(z) which maps  $\Omega$  onto the unit disk can be extended to a function which is
  - A) Analytic and one to one on  $\Omega \cap \gamma$
  - B) Analytic and one to one on  $\Omega \cup \gamma$
  - C) One to one and onto on  $\Omega \cap \gamma$
  - D) One to one and onto on  $\Omega\cup\gamma$
- 79. A real or complex function  $\varphi(t)$  of a real variable t, defined on an interval a < t < b, is said to be real analytic if, for every  $t_0$  in the interval, the Taylor development of  $\varphi(t)$ 
  - A) Converges in some interval  $(t_0 \rho, t_0 + \rho), \rho > 0$ .
  - B) Converges in some interval  $(t_0 \rho, t_0 + \rho), \rho < 0$ .
  - C) Converges in some interval  $(t_0 \rho, t_0 + \rho), \rho = 0$ .
  - D) None of the above
- 80. An analytic arc  $\varphi(t)$  is regular if
  - A)  $\varphi'(t) = 0$
  - B)  $\varphi'(t) \neq 0$
  - C)  $\varphi'(t) > 0$
  - D)  $\varphi'(t) < 0$
- 81. If every point of  $\gamma$  has a neighbourhood whose intersection with the whole boundary  $\partial \Omega$  is the same as its intersection with  $\gamma$ , then  $\gamma$  is a
  - A) One sided boundary arc
  - B) Two-sided boundary arc
  - C) Free boundary arc
  - D) None of these
- 82.  $e^z$  has a period
  - A) 2π
  - B) πi
  - C) 2*π*i
  - D) None of these
- 83. If  $\omega$  is a period so are all
  - A)  $n\omega$ , n is an integer
  - B)  $\omega/n$ , n is an integer
  - C)  $\omega^n$ , n is an integer
  - D) None of these
- 84. Sin z and Cos z are simply periodic functions with period
  - A) 3π
  - B) 2π
  - C) 0
  - D) 2πi
- 85. The doubly periodic meromorphic function is known as
  - A) Parabolic fuction
  - B) Elliptic function
  - C) Circular function
  - D) None of these
- 86. An elliptic function without poles reduces to
  - A) A constant
  - B) zero
  - C) 2πi
  - D) None of these
- 87. The sum of the residues of an elliptic function is
  - A) 2πi

B) infinity

C) 1

D) 0

- 88. An elliptic function with double pole at the origin as the only singularity is called the A) Weirstrass  $\rho$  function
  - B) Weirstrass  $\sigma$  function
  - C) Weirstrass  $\zeta$  function
  - D) None of these

89. The Weirstrass  $\zeta$  function is

- A) An even function
- B) An odd function
- C) Neither even nor odd
- D) None of these
- 90. The Weirstrass  $\zeta$  function is
  - A)  $\zeta'(z) = -\rho(z)$

B) 
$$\zeta(z) = -\rho'(z)$$

D)  $\zeta(z) = -\rho'(z)$ C)  $\zeta'(z) = \rho(z)$ 

D) 
$$\zeta(z) = \rho'(z)$$

91. The simple pole of Weirstrass  $\zeta$  function is

- A) Z=0 with residue 2
- B) Z=2 with residue 1
- C) Z=0 with residue 1
- D) Z=1 with residue 0

92. The Weirstrass  $\sigma$  function is defined as

A) 
$$\sigma(z) = z \prod_{\omega \neq 0} (1 - \frac{z}{\omega}) e^{\frac{z}{\omega} + \frac{(\frac{z}{\omega})^2}{2}}$$
  
B) 
$$\sigma(z) = z \prod_{\omega \neq 0} (1 + \frac{z}{\omega}) e^{\frac{z}{\omega} + \frac{(\frac{z}{\omega})^2}{2}}$$
  
C) 
$$\sigma(z) = z \prod_{\omega \neq 0} (1 - \frac{z}{\omega}) e^{\frac{z}{\omega} - \frac{(\frac{z}{\omega})^2}{2}}$$
  
D) 
$$\sigma(z) = z \prod_{\omega \neq 0} (1 + \frac{z}{\omega}) e^{\frac{z}{\omega} - \frac{(\frac{z}{\omega})^2}{2}}$$

93. 
$$z = 0$$
 and  $z = \omega$  are the

- A) Poles of  $\sigma(z)$
- B) Zeros of  $\sigma(z)$
- C) Removable singularities of  $\sigma(z)$
- D) Essential singularities of  $\sigma(z)$
- 94. The Weirstrass  $\sigma$  function is
  - A) An even function
  - B) An odd function
  - C) Neither even nor odd
  - D) None of these

95.  $\lim_{z \to 0} \frac{\sigma(z)}{z} =$ A) 0 **B**) 1 C) 2

- D) 3
- 96.  $\lim_{z \to 0} \frac{\sigma'(z)}{\sigma(z)} =$ 
  - A)  $\rho(z)$ 
    - B)  $\rho'(z)$
  - C)  $\zeta(z)$

D) 
$$\zeta'(z)$$

97. The Weirstrass  $\rho$  function can be represented as

A)  $\rho(z) = \frac{1}{z^2} + \sum_{\omega \neq 0} (\frac{1}{(z-\omega)^2} - \frac{1}{\omega^2})$  where the sum ranges over all  $\omega = n_1 \omega_1 - \frac{1}{\omega^2}$  $n_2\omega_2$  except 0.

- B)  $\rho(z) = \frac{1}{z^2} \sum_{\omega \neq 0} \left( \frac{1}{(z-\omega)^2} \frac{1}{\omega^2} \right)$  where the sum ranges over all  $\omega = n_1 \omega_1 + n_2 \omega_2$  except 0.
- C)  $\rho(z) = \frac{1}{z^2} \sum_{\omega \neq 0} \left( \frac{1}{(z-\omega)^2} \frac{1}{\omega^2} \right)$  where the sum ranges over all  $\omega = n_1 \omega_1 n_2 \omega_2$  except 0.
- D)  $\rho(z) = \frac{1}{z^2} + \sum_{\omega \neq 0} \left( \frac{1}{(z-\omega)^2} \frac{1}{\omega^2} \right)$  where the sum ranges over all  $\omega = n_1 \omega_1 + n_2 \omega_2$  except 0.
- 98. A function is said to be doubly periodic if there exist exactly two nonzero
  - A) dependent elements  $\omega_1$  and  $\omega_2$  such that  $f(z + \omega_1) = f(z)$  and  $f(z + \omega_2) = f(z)$
  - B) independent elements  $\omega_1$  and  $\omega_2$  such that  $f(z + \omega_1) = f(z)$  and  $f(z + \omega_2) = f(z)$
  - C) independent elements  $\omega_1$  and  $\omega_2$  such that  $f'(z + \omega_1) = f'(z)$  and  $f'(z + \omega_2) = f'(z)$
  - D) dependent elements  $\omega_1$  and  $\omega_2$  such that  $f'(z + \omega_1) = f'(z)$  and  $f'(z + \omega_2) = f'(z)$
- 99. A nonconstant elliptic function has equally many poles as it has
  - A) zeros
  - B) essential singularity
  - C) removable singularity
  - D) None of the above
- 100. Legendre's relation for Weierstrass  $\zeta$  function is
  - A)  $\eta_1 \omega_1 + \eta_2 \omega_2 = 2\pi i$
  - B)  $\eta_1 \omega_1 \eta_2 \omega_2 = 2\pi i$
  - C)  $\eta_1 \omega_2 + \eta_2 \omega_1 = 2\pi i$
  - D)  $\eta_1\omega_2 \eta_2\omega_1 = 2\pi i$

## Answers

1. B

- 2. A
- B
   A
- 4. A 5. A
- 6. D
- 0. D 7. C
- 8. A
- 9. A
- 10. A
- 11. A
- 12. D
- 13. B 14. C
- 14. C 15. D
- 16. C
- 17. C
- 18. A
- 19. C
- 20. D 21. C
- 21. C 22. A
- 23. C
- 24. B
- 25. A

•	
26.	A
27	C
27.	c
28.	C
29	D
$\frac{2}{2}$	
30.	A
31	А
20	
32.	A
33.	С
24	C
54.	C
35.	С
26	D
50.	D
37.	А
38	Δ
50.	n D
39.	В
40	C
4.1	
41.	А
42	А
12	
43.	A
44.	Α
15	٨
45.	A
46.	В
17	C
+/.	Ċ
48.	A
<u>4</u> 9	Δ
50	
50.	A
51.	А
51.	
52.	A
53.	В
51	۸
54.	А
54. 55.	A A
54. 55. 56	A A C
54. 55. 56.	A A C
54. 55. 56. 57.	A A C B
54. 55. 56. 57. 58	A A C B B
54. 55. 56. 57. 58.	A A C B B
54. 55. 56. 57. 58. 59.	A A C B B C
54. 55. 56. 57. 58. 59. 60.	A A C B B C A
54. 55. 56. 57. 58. 59. 60.	A A C B B C A
<ul> <li>54.</li> <li>55.</li> <li>56.</li> <li>57.</li> <li>58.</li> <li>59.</li> <li>60.</li> <li>61.</li> </ul>	A A C B B C A A
<ul> <li>54.</li> <li>55.</li> <li>56.</li> <li>57.</li> <li>58.</li> <li>59.</li> <li>60.</li> <li>61.</li> <li>62.</li> </ul>	A A C B B C A A A
<ul> <li>54.</li> <li>55.</li> <li>56.</li> <li>57.</li> <li>58.</li> <li>59.</li> <li>60.</li> <li>61.</li> <li>62.</li> <li>63</li> </ul>	A A C B B C A A A C
<ul> <li>54.</li> <li>55.</li> <li>56.</li> <li>57.</li> <li>58.</li> <li>59.</li> <li>60.</li> <li>61.</li> <li>62.</li> <li>63.</li> </ul>	A A C B B C A A A C
<ul> <li>54.</li> <li>55.</li> <li>56.</li> <li>57.</li> <li>58.</li> <li>59.</li> <li>60.</li> <li>61.</li> <li>62.</li> <li>63.</li> <li>64.</li> </ul>	A A C B B C A A C B
<ul> <li>54.</li> <li>55.</li> <li>56.</li> <li>57.</li> <li>58.</li> <li>59.</li> <li>60.</li> <li>61.</li> <li>62.</li> <li>63.</li> <li>64.</li> <li>65.</li> </ul>	A A C B B C A A C B B B C B B
<ul> <li>54.</li> <li>55.</li> <li>56.</li> <li>57.</li> <li>58.</li> <li>59.</li> <li>60.</li> <li>61.</li> <li>62.</li> <li>63.</li> <li>64.</li> <li>65.</li> </ul>	A A C B B C A A A C B B B
<ul> <li>54.</li> <li>55.</li> <li>56.</li> <li>57.</li> <li>58.</li> <li>59.</li> <li>60.</li> <li>61.</li> <li>62.</li> <li>63.</li> <li>64.</li> <li>65.</li> <li>66.</li> </ul>	A A C B B C A A C B B A
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67.	A A C B B C A A C B B A B B A B
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 67.	A A C B B C A A C B B A C B B A B C A A C C B C A C B B C A C B B C A C B B C A C B B C A C B B C A C B B C A C B B C A C B B C A C B B C A A C B B B C A A C B B C A C B B C A C B B C A C A
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68.	A A C B B C A A C B B A B C B B A C C C C
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 64. 65. 66. 67. 68. 69.	A A C B B C A A C B B A B C B B A B C B B C B B C A C B B C A C B B C A C B B C A C B B C A C B B C A C B B C A C B B C A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C B B C A A C B B B C A A C B B B C A C B B C A A C B B C A A C B B B C A A B B B C A A C B B B C A A C B B B B
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70	A A C B B C A A C B B A B C B A B C B A C B B C A A C B B B C A A C B B C A A C B B C A A C B B B C A A C B B A C B B B C A A C B B B C A A C B B B C A A C B B B B
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70.	A A C B B C A A C B B A B C B A B C B A C B B C A A C B B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A C B B C A C B B C A C A
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71.	A A C B B C A A C B B A B C B A B C B A A C B B C A C A
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72	A A C B B C A A C B B A B C B A A B C B A A B C A A C B B C A A A C B B C A A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B C
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72.	A A C B B C A A C B B A B C B A A B C B A A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B C
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73.	A A C B B C A A C B B A B C B A A B C B A A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B A C B B B C B A C B B C B B C B B C B A C B B B C B B C B B C B C
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74	A A C B B C A A C B B A B C B A A B C B A A C B B C A A C B A C B B C B A C B B C B A C B B C B A C B A C B B C B A C B B C B A C B B C B A C B B B C B C
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75.	A A C B B C A A C B B A B C B A A B C B A A C B B C B A C B B C A A C B B C B C
<ul> <li>54.</li> <li>55.</li> <li>56.</li> <li>57.</li> <li>58.</li> <li>59.</li> <li>60.</li> <li>61.</li> <li>62.</li> <li>63.</li> <li>64.</li> <li>65.</li> <li>66.</li> <li>67.</li> <li>68.</li> <li>69.</li> <li>70.</li> <li>71.</li> <li>72.</li> <li>73.</li> <li>74.</li> <li>75.</li> </ul>	A A C B B C A A A C B B A B C B A A B A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C B A C B B C B A C B B C B B C B B C B B C B B C B B C B B C B B C B B C B B C B B C B B C B B C B B B C B B C B B C B B C B B C B B C B B C B B C B B C B B B B C B B C B B B B C B B C B B C B B C B B B C B B C B B C B B C B B C B B C B B C B B C B B C B B C B B C B B C B B C B B C B B C B B C B B B C B B C B B C B B C B B C B B C B B C B B B C B B B C B B C B B B C B B B C B B B C B B C B B B C B B B B C B B C B B B B B B C B B B B B C B
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 64. 65. 66. 67. 70. 71. 72. 73. 74. 75. 76.	A A C B B C A A A C B B A B C B A A B A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C B A C B B C B A C B B C B C
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 64. 65. 66. 67. 70. 71. 72. 73. 74. 75. 76. 77.	A A C B B C A A A C B B A B C B A A B C C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C B A C B B C B A C B B C B A C B B C B B C B B C B C
$\begin{array}{c} 54.\\ 55.\\ 56.\\ 57.\\ 58.\\ 59.\\ 60.\\ 61.\\ 62.\\ 63.\\ 64.\\ 65.\\ 66.\\ 67.\\ 68.\\ 69.\\ 70.\\ 71.\\ 73.\\ 74.\\ 75.\\ 76.\\ 77.\\ 76.\\ 77.\\ \end{array}$	A A C B B C A A C B B A B C B A A B C B A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C B A C B B C B B C B B C B B C B B C B C
$\begin{array}{c} 54.\\ 55.\\ 56.\\ 57.\\ 58.\\ 59.\\ 60.\\ 61.\\ 62.\\ 63.\\ 64.\\ 65.\\ 66.\\ 67.\\ 68.\\ 69.\\ 70.\\ 71.\\ 73.\\ 74.\\ 75.\\ 76.\\ 77.\\ 78. \end{array}$	A A C B B C A A A C B B A B C B A A B A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B C
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79.	A A C B B C A A A C B B A B C B A A B A C B B C A A A C B B C A A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C B A C B B C B A C B B C B A C B B C B C
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79.	A A C B B C A A A C B B A B C B A A B C B A A C B B C A A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B B C B A C B B C B B C B A C B B C B C
<ul> <li>54.</li> <li>55.</li> <li>56.</li> <li>57.</li> <li>58.</li> <li>59.</li> <li>60.</li> <li>61.</li> <li>62.</li> <li>63.</li> <li>64.</li> <li>65.</li> <li>66.</li> <li>67.</li> <li>68.</li> <li>69.</li> <li>70.</li> <li>71.</li> <li>72.</li> <li>73.</li> <li>74.</li> <li>75.</li> <li>76.</li> <li>77.</li> <li>78.</li> <li>79.</li> <li>80.</li> </ul>	A A C B B C A A A C B B A B C B A B C B A A C B B C A A A C B B C A A A C B B C A A A C B B C A A A C B B C A A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C A A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B A C B B C B C
<ul> <li>54.</li> <li>55.</li> <li>56.</li> <li>57.</li> <li>58.</li> <li>59.</li> <li>60.</li> <li>61.</li> <li>62.</li> <li>63.</li> <li>64.</li> <li>65.</li> <li>66.</li> <li>67.</li> <li>68.</li> <li>69.</li> <li>70.</li> <li>71.</li> <li>72.</li> <li>73.</li> <li>74.</li> <li>75.</li> <li>76.</li> <li>77.</li> <li>78.</li> <li>79.</li> <li>80.</li> <li>81</li> </ul>	A A C B B C A A A C B B A B C B A A B A C B D B B A B C
54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 80. 81. 82.	A A C B B C A A A C B B A B C B A A B A C B D B B A B C C

83. A	
84. B	
85. B	
86. A	
87. D	
88. A	
89. B	
90. A	
91. C	
92. A	
93. B	
94. B	
95. B	
96. C	
97. D	
98. B	
99. A	
100.	D