Measure Theory MCQ -Module 1

1. If A and B are two sets such that A ⊆ B, then
   m*(A) ≤ m*(B) (B) m* (A) ≥ m*(B) (C) m* (A) ≠ m*(B) (D) None of these
2. Which of the following statements is false for a Lebesgue Outer Measure of an arbitrary set
   (A) m* (A) ≥ 0 ∀ A
   (B) If A ⊆ B, then m* (A) ≤ m*(B)
   (C) m* (A+x) = m* (A) ∀ x ϵ R
   (D) If {E_k } is any countable collection of sets disjoint or not , then
   m*($\bigcup_{k=1}^{\infty}$ E_k) > $\sum_{k=1}^{\infty} m^*(E_k)$
3. A set E is said to be measurable if for each set A
   (A) m* (A) = m* (A∩E) + m* (A ∩ E^c)
   (B) m* (A)= m* (A U E) + m* (A U E^c)
   (C) m* (E) = m* (A∩E) + m* (A ∩ E^c)
   (D) m* (E)= m* (A U E) + m* (A U E^c)
4. Identify the wrong statement
   (A) cantor set is uncountable
   (B) cantor set is open
   (C) cantor set is measurable
   (D) cantor set is nonempty

5. Which property is called countable subadditivity of outer measure
   (A) m*($\bigcup_{k=1}^{\infty}$ E_k) ≤ $\sum_{k=1}^{\infty} m^*(E_k)$
   (B) m*($\bigcup_{k=1}^{\infty}$ E_k) ≥ $\sum_{k=1}^{\infty} m^*(E_k)$
   (C) m*($\bigcup_{k=1}^{n} E_k$) ≥ $\sum_{k=1}^{n} m^*(E_k)$
   (D) None of these

6. Which is a Measurable set?
   (A) open set
   (B) closed set
   (C) Borel set
   (D) All the above

7. The outer measure of a countable set is
   (A) 0
   (B) ∞
   (C) 1
   (D) Not defined

8. The outer measure of Cantors set is
   (A) ∞ (B) 1 (C) 0 (D) $\frac{1}{3}$

9. If $\{E_k\}_{k=1}^{\infty}$ is an ascending collection of measurable sets and $E = \bigcup_{k=1}^{\infty} E_k$, then m(E) is equal to

(A) $\sum_{k=1}^{\infty} m(E_k)$ (B) 0 (C) ∞ (D) $\lim_{k\to\infty} m(E_k)$

10. If $\{E_k\}_{k=1}^{\infty}$ is an descending collection of measurable sets with m (E_1) < ∞ and E= $\bigcap_{k=1}^{\infty} E_k$, then m(E) is equal to

(A) $\sum_{k=1}^{\infty} m(E_k)$ (B) 0 (C) ∞ (D) $\lim_{k\to\infty} m(E_k)$

11. Which of the following statements is true for a Lebesgue measure

(A) countable additivity

(B) Monotonicity

(C) translation invariant

(D) All the above are true

12. If I =[a,b] is a closed interval, then length of I is

(A) a+b (B) a-b (C) b-a (D) None of these

13. Which of the following statement is true

(A) There exists a non-measurable subset of real numbers

(B) There exists a measurable set which is not a Borel set.

(C) There exists an uncountable set having measure zero.

(D) All the above are true

14. Which of the following is an algebra on X={1,2,3}

(A) {Ø}

(B) {Ø , {1}, {1,2}}

(C) {Ø, {1}, {2,3}, {1,2,3}}

(D){X}

15. If A and B are two sets such that m* (A) =0, then

(A) m* (AUB) = m* (B)

(B) m* (A) = m* (B)

(C) m* (A) ≤ m* (B)

(D) m* (B) ≤ m* (A)

16. Which of the following statement is true

(A) F_σ set is measurable.

(B) G_δ set is measurable.

(C) Borel set is measurable

(D)All the above are true

17. The outer measure of empty set is

(A) 0 (B) ∞ (C) 1 (D) Ø

18. m* ({k}) = ……., where k is a real number

(A) 1 (B) 0 (C) k (D) None of these

19. Identify the correct statement

(A) Cantor – Lebesgue function is an increasing function

(B) Cantor – Lebesgue function is a continuous function

(C) Cantor – Lebesgue function maps [0,1] onto [0,1]

(D) All the above are true.

20. If Ω is an algebra and A, B ϵ Ω, then

(A) A-B ϵΩ (B) AΔB ϵ Ω (C) A∩B ϵ Ω (D)All the above are true

21. F_σ set is the

(A) intersection of a countable collection of open sets.

(B) intersection of a countable collection of closed sets

(C) union of a countable collection of open sets

(D) union of a countable collection of closed sets

22. G_δ set is the

(A) intersection of a countable collection of open sets.

(B) intersection of a countable collection of closed sets

(C) union of a countable collection of open sets

(D) union of a countable collection of closed sets

23. Which one is not equivalent to the measurability of any set of real numbers E

(A) For each ϵ>0, there is an open set O containing E for which m* (O-E)< ϵ

(B) For each ϵ>0, there is a closed set F containing E for which m* (F-E)< ϵ

(C) For each ϵ>0, there is a closed set F contained in E for which m* (E-F)< ϵ

(D) There is a G_δ set G containing E for which m* (G-E)< ϵ

24. For any set E and any ϵ>0,there exists an open set O containing E for which

(A) m* (O)≥ m* (E) + ϵ

(B) m* (O)≤ m * (E) - ϵ

(C) m* (O) ≥ m* (E) - ϵ

(D)m* (O)≤ m* (E) + ϵ

25. Let A be the set of irrational numbers in [0,1], then

(A) m* (A)= 0

(B) m* (A) =1

(C) m* (A)= ∞

(D) m* (A)= 2

Measure Theory Question Bank: Module 2

August 9, 2022

1. $\{f_n\}$ be a sequence of increasing functions on a set E, if for all index n:
(A) $f_n \geq f_{n+1}$
(B) $f_n \leq f_{n+1}$
(C) $f_n \neq f_{n+1}$
(D) $f_n = f_{n+1}$

2. Let f be an extended real valued function defined on a measurable domain E. If f is Lesbesgue measurable, then for each real number c, which of the following statements is true.
(A) $\{x \in E | f(x) > c\}$ is measurable.
(B) $\{x \in E | f(x) < c\}$ is measurable.
(C) $\{x \in E | f(x) = c\}$ is measurable.
(D) All of the above.

3. Let f be an extended real valued function defined on a measurable domain E. Then f is measurable if and only if for each open set O, $f^{-1}(O) = \{x \in E | f(x) \in O\}$ is:
(A) $f^{-1}(O) = \{x \in E | f(x) \in O\}$ is open.
(B) $f^{-1}(O) = \{x \in E | f(x) \in O\}$ is measurable.
(C) $f^{-1}(O) = \{x \in E | f(x) \in O\}$ is closed.
(D) None of the above.

4. Which one of the following is false.
(A) A monotone function that is defined on an interval is not measurable.
(B) Let f be an extended measurable real-valued function on E and $f = g$ a.e. on E, then g is measurable on E.
(C) A monotone function that is defined on an interval is measurable.
(D) A real-valued function that is continuous on its measurable domain is measurable.

5. Let f and g be measurable functions on E that are finite a.e. on E. Then which of the following statements is false.
(A) For $\alpha, \beta \in \mathbb{R}$, $\alpha f + \beta g$ is measurable on E.
(B) $(f+g)^2$ is measurable on E.
(C) fg is measurable on E.
(D) None of the above.

6. If A is any set. Then the characteristic function χ_A is defined as
(A) $\chi_A = \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{otherwise.} \end{cases}$

(B) $\chi_A = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise.} \end{cases}$

(C) $\chi_A = \begin{cases} 1 & \text{if } x \in A \\ -1 & \text{otherwise.} \end{cases}$

(D) None of the above.

7. The characteristic function χ_A is measurable if and only if
(A) A has measure 0.
(B) A has infinite measure.
(C) A is measurable.
(D) None of the above.

8. Which one of the following is false.
(A) If f is measurable on E, then $|f|$ is measurable.
(B) If f is measurable on E, then, f^- is not measurable.
(C) If f is measurable on E, then f^+ is measurable.
(D) If f is measurable on E, then, f^- is measurable.

9. A real-valued function ϕ defined on a measurable set E is simple if
(A) It is measurable.
(B) It assumes only a finite number of values.
(C) It is measurable and assumes only a finite number of values.
(D) None of the above.

10. A bounded real-valued function f defined on a closed, bounded interval $[a, b]$ is Riemann integrable over $[a, b]$ if:
(A) $(R)\int_{\underline{a}}^{b} f < (R)\int_{a}^{\overline{b}} f$.
(B) $(R)\int_{\underline{a}}^{b} f > (R)\int_{a}^{\overline{b}} f$.
(C) $(R)\int_{\underline{a}}^{b} f \leq (R)\int_{a}^{\overline{b}} f$.
(D) $(R)\int_{\underline{a}}^{b} f = (R)\int_{a}^{\overline{b}} f$.

11. Dirichlet's function f is defined on $[0, 1]$ as:

(A) $f(x) = \begin{cases} 0 & \text{if } x \text{ rational} \\ 1 & \text{if } x \text{ irrational.} \end{cases}$

(B) $f(x) = \begin{cases} 0 & \text{if } x \leq 0.5 \\ 1 & \text{if } x > 0.5 \end{cases}$

(C) $f(x) = \begin{cases} 1 & \text{if } x \text{ rational} \\ 0 & \text{if } x \text{ irrational.} \end{cases}$

(D) None of the above.

12. Which of the following is the Reimann integral of the Dirichlet's function defined on $[0, 1]$?
(A) 1
(B) 0
(C) Does not exist.
(D) None of the above.

13. Let f be a bounded function defined on $[a, b]$. Then which of the following statements is correct:
(A) If f is Lesbesgue integrable over $[a, b]$, then f is Reimann integrable over $[a, b]$.
(B) If f is Reimann integrable over $[a, b]$, then f is Lesbesgue integrable over $[a, b]$.
(C) If f is Reimann integrable over $[a, b]$, then f may not be Lesbesgue integrable over $[a, b]$.
(D) None of the above.

14. Let f be a bounded measurable function on a set of finite measure E. Suppose A and B are disjoint measurable subsets of E. Then $\int_{A \cup B} f =$
(A) $\int_A f + \int_B f$.
(B) $\int_A f - \int_B f$.
(C) $\int_A f + \int_B f - \int_{A \cap B} f$.
(D) None of the above.

15. Let f be a bounded measurable function on a set of finite measure E. Then
(A) $\left|\int_E f\right| > \int_E |f|$.
(B) $\left|\int_E f\right| = \int_E |f|$.
(C) $\left|\int_E f\right| \geq \int_E |f|$.
(D) $\left|\int_E f\right| \leq \int_E |f|$.

16. Let f be a nonnegative measurable function on E . Then for any $\lambda > 0$,
(A) $m\{x \in E | f(x) \geq \lambda\} \leq \frac{1}{\lambda} \int_E f$.
(B) $m\{x \in E | f(x) \geq \lambda\} \geq \frac{1}{\lambda} \int_E f$.
(C) $m\{x \in E | f(x) \geq \lambda\} = \frac{1}{\lambda} \int_E f$.
(D) $m\{x \in E | f(x) \geq \lambda\} > \frac{1}{\lambda} \int_E f$.

17. Let f be a nonnegative measurable function on E. Then $\int_E f = 0$ if and only if
(A) $f > 0$ a.e. on E.
(B) $f = 0$ a.e. on E.
(C) $f \leq 0$ a.e. on E.
(D) $f < 0$ a.e. on E.

18. Let $\{f_n\}$ be a sequence of nonnegative measurable functions on E. If $\{f_n\} \to f$ pointwise a.e. on E, then
(A) $\int_E f = \liminf \int_E f_n$.
(B) $\int_E f \geq \liminf \int_E f_n$.
(C) $\int_E f \leq \liminf \int_E f_n$.
(D) None of the above.

19. Let $E = \mathbb{R}$. For any natural number n, define $g_n = \chi_{(n,n+1)}$. Then g_n converges pointwise to
(A) $g \equiv 0$ on E.
(B) $g \equiv 0.5$ on E.
(C) $g \equiv 1$ on E.
(D) None of the above.

20. A nonnegative measurable function f on a measurable set E is said to be integrable over E if
(A) $\int_E f = 0$.
(B) $\int_E f < \infty$.
(C) $\int_E f = \infty$.
(D) None of the above.

21. Let the nonnegative function f be integrable over E. Then
(A) f is constant a.e. on E.
(B) f is 0 a.e. on E.
(C) f is finite a.e. on E.
(D) None of the above.

22. For an extended real-valued function f on E, the positive part f^+ of f is:
(A) $f^+(x) = \max\{f(x), 0\}$.
(B) $f^+(x) = \max\{-f(x), 0\}$.
(C) $f^+(x) = -\max\{f(x), 0\}$.
(D) $f^+(x) = \min\{f(x), 0\}$.

23. For an extended real-valued function f on E, the negative part f^- of f is:
(A) $f^-(x) = \min\{-f(x), 0\}$.
(B) $f^-(x) = \max\{-f(x), 0\}$.
(C) $f^-(x) = -\max\{f(x), 0\}$.
(D) None of the above.

24. If $|f|$ is integrable over E, then the integral of f over E is given by
(A) $\int_E f = \int_E f^+ + \int_E f^-$.
(B) $\int_E f = \int_E f^+ - \int_E f^-$.
(C) $\int_E f = \int_E f^- - \int_E f^+$.
(D) None of the above.

25. Let the functions f and g be integrable over E. Then for any α and β,
(A) $\int_E(\alpha f + \beta g) = \alpha \int_E f - \beta \int_E g$.
(B) $\int_E(\alpha f + \beta g) = -\alpha \int_E f + \beta \int_E g$.
(C) $\int_E(\alpha f + \beta g) = -\alpha \int_E f - \beta \int_E g$.
(D) $\int_E(\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$.

Measure Theory MCQ-Module 3

1. A pair $(X, \mathcal{M})$ consisting of a set X and a σ-algebra $\mathcal{M}$ of subsets of X is called a
   (A) Measurable space
   (B) σ-finite space
   (C) Measure Space
   (D) Complete Measure Space

2. Let $(X, \mathcal{M}, \mu)$ be a measure space. A subset E of X is called measurable if
   (A) $\mu(E) = 0$
   (B) $\mu(E) \neq 0$
   (C) $E \in \mathcal{M}$
   (D) $E \notin \mathcal{M}$

3. A measurable space $(X, \mathcal{M})$ together with a measure μ defined on it, is called a
   (A) Topological Space
   (B) Measure Space
   (C) σ-finite space
   (D) Complete Measure Space

4. If μ is a measure on a measurable space $(X, \mathcal{M})$, which of the following statements is true.
   (A) $\mu: \mathcal{M} \to [0, \infty]$
   (B) $\mu(\emptyset) = 0$
   (C) μ is countably additive
   (D) All the above

5. For any set X, we define $\mathcal{M} = 2^X$, the collection of all subsets of X, and define a measure η by defining the measure of a finite set to be the number of elements in the set and the measure of an infinite set to be ∞. This measure is called
   (A) Counting measure
   (B) Lebesgue measure

(C) Dirac Measure

(D) Borel Measure

6. For any σ-algebra $\mathcal{M}$ of subsets of a set X and point x_0 belonging to X, we can define a measure δ_{x_0} by

$$\delta_{x_0}(E) = \begin{cases} 1 & \text{if } x_0 \in E \\ 0 & \text{if } x_0 \notin E \end{cases} \quad \text{for every } E \in \mathcal{M}$$

This measure is called

(A) Counting measure

(B) Lebesgue measure

(C) Dirac Measure

(D) Borel Measure

7. Let $(X, \mathcal{M}, \mu)$ be a measure space. Then for any finite disjoint collection $\{E_k\}_{k=1}^{n}$ of measurable sets,

$$\mu\left(\bigcup_{k=1}^{n} E_k\right) = \sum_{k=1}^{n} \mu(E_k)$$

This property is known as

(A) Monotonicity

(B) Finite Additivity

(C) Excision

(D) Continuity of Measure

8. Let $(X, \mathcal{M}, \mu)$ be a measure space. If A and B are measurable sets and $A \subseteq B$, then

$$\mu(A) \leq \mu(B)$$

This property is known as

(A) Finite Additivity

(B) Excision

(C) Monotonicity

(D) Continuity of Measure

9. Let $(X, \mathcal{M}, \mu)$ be a measure space. If A and B are measurable sets, $A \subseteq B$ and $\mu(A) < \infty$, then

$$\mu(B \sim A) \leq \mu(B) - \mu(A)$$

This property is known as

(A) Finite Additivity

(B) Excision

(C) Monotonicity

(D) Continuity of Measure

10. Let $(X, \mathcal{M}, \mu)$ be a measure space. Then for any countable collection $\{E_k\}_{k=1}^{\infty}$ of measurable sets that covers a measurable set E,

$$\mu(E) \leq \sum_{k=1}^{\infty} \mu(E_k)$$

This property is known as

(A) Monotonicity

(B) Countable additivity

(C) Continuity of Measure

(D) Countable Monotonicity

11. Let $(X, \mathcal{M}, \mu)$ be a measure space. Then for any countable disjoint collection $\{E_k\}_{k=1}^{\infty}$ of measurable sets,

$$\mu\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} \mu(E_k)$$

This property is known as

(A) Countable Additivity

(B) Monotonicity

(C) Countable Monotonicity

(D) Continuity of Measure

12. A sequence of sets $\{E_k\}_{k=1}^{n}$ is called ascending if

(A) for each k, $E_{k+1} = E_k$

(B) for each k, $E_{k+1} \neq E_k$

(C) for each k, $E_k \subseteq E_{k+1}$

(D) for each k, $E_{k+1} \subseteq E_k$

13. Let $(X, \mathcal{M}, \mu)$ be a measure space. If $\mu(X) < \infty$, then the measure μ is said to be

(A) Positive

(B) Finite

(C) Semifinite

(D) σ-finite

14. Let $(X, \mathcal{M}, \mu)$ be a measure space. If X is the union of a countable collection of measurable sets, each of which has finite measure, then the measure μ is said to be

(A) Positive

(B) Finite

(C) Semifinite

(D) σ-finite

15. Let $(X, \mathcal{M}, \mu)$ be a measure space. A measurable set E is said to be of finite measure provided

(A) $\mu(E) < \infty$

(B) $\mu(E) = \infty$

(C) $\mu(E) \geq 0$

(D) $\mu(E) = 0$

16. Let $(X, \mathcal{M}, \mu)$ be a measure space and E is a measurable set. If E is the union of a countable collection of measurable sets, each of which has finite measure, then E is said to be

(A) Finite

(B) σ-finite

(C) positive

(D) saturated

17. Which of the following is a finite measure?

(A) Counting measure on an uncountable set

(B) Lebesgue measure on $(-\infty, \infty)$

(C) Lebesgue measure on $[0,1]$

(D) None of these

18. A measure space $(X, \mathcal{M}, \mu)$ is said to be complete provided

(A) $\mathcal{M}$ contains all subsets of X

(B) $\mathcal{M} = \{X, \emptyset\}$

(C) $\mathcal{M}$ contains all subsets of sets of measure ∞

(D) $\mathcal{M}$ contains all subsets of sets of measure zero

19. Let ν be a signed measure. A set A is called positive with respect to ν if

(A) A is measurable and $\nu(E) \geq 0$ for all measurable subsets E of A.

(B) A is measurable and $\nu(E) > 0$ for all measurable subsets E of A.

(C) A is measurable and $\nu(A) > 0$.

(D) A is measurable and $\nu(A) \geq 0$

20. Let ν be a signed measure. A set B is called negative with respect to ν if

(A) B is measurable and $\nu(B) < 0$.

(B) B is measurable and $\nu(B) \leq 0$

(C) B is measurable and $\nu(E) \leq 0$ for all measurable subsets E of B.

(D) B is measurable and $\nu(E) < 0$ for all measurable subsets E of B.

21. Let ν be a signed measure. A set A is called null with respect to ν provided

(A) A is measurable and $\nu(A) = 0$.

(B) A is measurable and $\nu(A) \geq 0$

(C) A is measurable and $\nu(E) \geq 0$ for all measurable subsets E of A.

(D) A is measurable and $\nu(E) = 0$ for all measurable subsets E of A.

22. Let ν be a signed measure on the measurable space $(X, \mathcal{M})$. Which of the following is a Hahn decomposition for ν?

(A) $X = A \cup B$ where $A \cap B \neq \emptyset$, A is positive for ν and B is negative for ν

(B) $X = A \cup B$ where $A \cap B = \emptyset$, A is positive for ν and B is negative for ν

(C) $X = A \cup B$ where $A \cap B = \emptyset$ and both A and B are positive for ν

(D) $X = A \cup B$ where $A \cap B = \emptyset$ A is positive for ν and B is negative for ν

23. Let v be a signed measure on the measurable space $(X, \mathcal{M})$. Which of the following gives the Jordan decomposition of v?

(A) $\nu = \nu^+ - \nu^-$ where $\nu^+$and ν^- are mutually singular measures on $(X, \mathcal{M})$

(B) $\nu = \nu^+ + \nu^-$ where $\nu^+$and ν^- are mutually singular measures on $(X, \mathcal{M})$

(C) $\nu = \nu^+ - \nu^-$ where $\nu^+$and ν^- are measures on $(X, \mathcal{M})$

(D) $\nu = \nu^+ + \nu^-$ where $\nu^+$and ν^- are measures on $(X, \mathcal{M})$

24. Which of the following statements defines an outer measure on a set X.

(A) $\mu^*: 2^X \to [-\infty, \infty]$, $\mu^*(\emptyset) = 0$ and μ^* is countably monotone.

(B) $\mu^*: 2^X \to [0, \infty]$, $\mu^*(\emptyset) = 0$ and μ^* is countably monotone.

(C) $\mu^*: 2^X \to [-\infty, \infty]$, $\mu^*(\emptyset) = 0$ and μ^* is monotone.

(D) $\mu^*: 2^X \to [0, \infty]$, $\mu^*(\emptyset) = 0$ and μ^* is monotone.

25. For a set X and the σ-algebra $\mathcal{M} = \{X, \emptyset\}$, which functions are measurable with respect to $\mathcal{M}$.

(A) Only the constant functions on X

(B) Every extended real-valued function on X

(C) Only the Identity Function on X

(D) Only the simple functions on X

26. Let $(X, \mathcal{M})$ be a measurable space and f and g be any two measurable real-valued functions on X. Then which of the following statements is false.

(A) $f - g$ is measurable

(B) fg is measurable

(C) $f \circ g$ is measurable

(D) $\max(f, g)$ is measurable

27. Two measures v_1 and v_2 on a measurable space $(X, \mathcal{M})$ are said to be mutually singular if there are disjoint measurable sets A and B with $X = A \cup B$ for which

(A) $\nu_1(A) \geq 0$ and $\nu_2(B) \leq 0$

(B) $\nu_1(A) = \nu_2(B) = 0$

(C) $\nu_1(A) = 0$ and $\nu_2(B) \neq 0$

(D) $\nu_1(A) \neq 0$ and $\nu_2(B) \neq 0$

Measure Theory MCQ– Module 4

1. Which category of functions are used to define the integral of non-negative measurable functions?

   a) Simple functions b) Step functions

   c) Real valued functions d) Extended real valued functions

2. Let $(X, \mathcal{M}, \mu)$ be a measure space, f be a nonnegative measurable function on X, and λ is a positive real number, then the Chebychev's inequality states that:

   a) $\mu\{x \in X / f(x) > \lambda\} \leq \frac{1}{\lambda} \int f \, d\, \mu$ b) $\mu\{x \in X / f(x) \leq \lambda\} < \frac{1}{\lambda} \int f \, d\, \mu$

   c) $\mu\{x \in X / f(x) > \lambda\} \leq \frac{1}{\lambda} \int f \, d\, \mu$ d) $\mu\{x \in X / f(x) \geq \lambda\} \leq \frac{1}{\lambda} \int f \, d\, \mu$

3. The Monotone Convergence Theorem is valid for

   a) Simple functions b) Step functions

   c) Non negative measurable functions d) Measurable functions

4. The convergence of a sequence of bounded integrals corresponding to a sequence of nonnegative integrable functions, to the integral of a function which is finite a.e. is guaranteed by

   a) Monotone convergence theorem b) Lebesgue's convergence theorem

   c) Fatou's lemma d) Beppo Levi's lemma

5. Let $(X, \mathcal{M}, \mu)$ be a measure space and f a nonnegative measurable function on X. Then f is said to be integrable over X with respect to μ if,

   a) $\int f \, d\mu \geq 0$ b) $\int f \, d\mu < \infty$

   c) $\int f \, d\mu \leq \infty$ d) $\int f \, d\mu > 0$

6. Let f be a non-negative measurable function on a space (X, μ) such that $f = 0$ a.e. on X. Then

   a) $\int_X f d\mu > 0$ b) $\int_X f d\mu \geq 0$

   c) $\int_X f d\mu \leq 0$ d) $\int_X f d\mu = 0$

7. Let f and g be nonnegative measurable functions on X for which $g \leq f$ a.e. on X. Then

a) $\int_X fd\mu \geq \int_X gd\mu$

b) $\int_X fd\mu > \int_X gd\mu$

c) $\int_X fd\mu = \int_X gd\mu$

d) $\int_X fd\mu \neq \int_X gd\mu$

8. The positive part of a measurable function is defined as

a) max $\{f, 0\}$

b) max $\{-f, 0\}$

c) max $\{f, -f\}$

d) max $\{+f, -f\}$

9. Let $(X, \mathcal{M}, \mu)$ be a measure space and f a measurable function on$X$. Then f is said to be integrable over X with respect to μ if,

a) only $f^+$is integrable

b) only $f^-$is integrable

c) either f^+ or f^- is integrable

d) $|f|$ is integrable.

10. Let $(X, \mathcal{M}, \mu)$ be a measure space and $\{f_n\}$, a sequence of integrable functions on X. Suppose for each $\varepsilon > 0$, there is a $\delta > 0$ such that for any natural number n and measurable set $E \subseteq X$, if $\mu(E) < \delta$, then $\int_E |f_n| d\mu < \infty$. Then

a) $\{f_n\}$ is pointwise integrable

b) $\{f_n\}$ is finitely integrable

c) $\{f_n\}$ is uniformly measurable

d) $\{f_n\}$ is uniformly integrable

11. Let $(X, \mathcal{M}, \mu)$ be a measure space for which $\mu(x) = 0$ and $f = \infty$ on X. Then,

a) $\int_X f\, d\mu \geq 0$

b) $\int_X f\, d\mu = 0$

c) $\int_X f\, d\mu = \infty$

d) $\int_X f\, d\mu < \infty$

12. Which theorem establishes the passage of limit under the integral sign in the case of a sequence of integrable functions converging pointwise?

a) Monotone convergence theorem

b) Lebesgue dominated convergence theorem

c) Vitali convergence theorem

d) Bounded convergence theorem

13. Which theorem states that a σ −finite measure defined on a measurable space $(X, \mathcal{M})$ can be determined by the integral of a nonnegative measurable function?

a) Lebesgue decomposition theorem

b) Lebesgue convergence theorem

c) Vitali convergence theorem

d) Radon -Nikodym theorem

14. Two measures μ & ν on a measurable space $(X, \mathcal{M})$ are said to be mutually singular if there are disjoint sets A & B with,

a) $X = A \cup B$ & $\mu(A) = \nu(B)$ b) $X = A \cup B$ & $\mu(A) \neq \nu(B)$

c) $X = A \cup B$ & $\mu(A) \geq \nu(B)$ d) $X = A \cup B$ & $\mu(A) = \nu(B) = 0$

15. If μ & ν are measures on a measurable space $(X, \mathcal{M})$ such that $\mu \perp \nu$ and $\mu \ll \nu$, then

a) $\mu = 0$ b) $\nu = 0$

c) $\mu = \nu = 0$ d) $\mu \geq \nu$

16. If $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \nu)$ are two measure spaces **,** then $A \times B$ is called a measurable rectangle if

a) $A \subseteq X, B \subseteq Y$ & $A \notin \mathcal{A}, B \in \mathcal{B}$ b) $A \subseteq X, B \subseteq Y$ & $A \in \mathcal{A}, B \notin \mathcal{B}$

c) $A \subseteq X, B \subseteq Y$ & $A \notin \mathcal{A}, B \notin \mathcal{B}$ d) $A \subseteq X, B \subseteq Y$ & $A \in \mathcal{A}, B \in \mathcal{B}$

17. Let $E \subseteq X \times Y$, and f be a function on E. Then the x −section of E is a subset of

a) $X \times Y$ b) X

c) Y d) E

18. Let $\mathcal{R}$ be the collection of measurable rectangles in $X \times Y$ and for a measurable rectangle $A \times B$, define $\lambda(A \times B) = \mu(A).\nu(B)$. Then λ is called a,

a) premeasure b) outer measure

c) product measure d) induced measure

19. Let $\mathcal{R}$ be the collection of measurable rectangles in $X \times Y$. Then $\mathcal{R}$ is a,

a) ring b) group

c) semiring d) semigroup

20. Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \nu)$ be two measure spaces, $\{A_k \times B_k\}_{k=1}^{\infty}$ be a countable disjoint collection of measurable rectangles whose union is a measurable rectangle $A \times B$, then

a) $\mu(A) \times \nu(B) = \sum_{k=1}^{\infty} \mu(A_k) \times \nu(B_k)$ b) $\mu(A) \times \nu(B) < \sum_{k=1}^{\infty} \mu(A_k) \times \nu(B_k)$

c) $\mu(A) \times \nu(B) > \sum_{k=1}^{\infty} \mu(A_k) \times \nu(B_k)$ d) $\mu(A) \times \nu(B) \neq \sum_{k=1}^{\infty} \mu(A_k) \times \nu(B_k)$

21. If μ & ν are complete measures, then $\mu \times \nu$ is

a) always complete
b) not always complete
c) not at all complete
d) none of these

22. If $(X, \mathcal{A})$ and $(Y, \mathcal{B})$ are two measurable spaces , f is $\mathcal{A}-$measurable, g is $\mathcal{B}-$measurable, then, fg is,

a) $\mathcal{A}-$measurable
b) $\mathcal{B}-$measurable
c) $\mathcal{A} \times \mathcal{B}-$measurable
d) not measurable

23. Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \nu)$ be two measure spaces. Then Fubini's theorem is valid if

a) μ & ν are not complete
b) μ is complete
c) ν is not complete
d) ν is complete

24. Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \nu)$ be two measure spaces. Then Tonelli's theorem is valid if

a) only μ is $\sigma-$finite
b) both μ & ν are $\sigma-$finite
c) only ν if $\sigma-$finite
d) both μ & ν need not be $\sigma-$finite

25. Fubini's theorem and Tonelli's theorem deals with

a) iterated measures
b) iterated products
c) iterated integrals
d) iterated rectangles
