MCQ (FOR PRIVATE EXAMS) – SEM II- COMPLEX ANALYSIS

1. What is the maximum number of fixed points of a linear fractional transformation?
   a) One
   b) Two
   c) Three
   d) Four
2. Identify the fixed points of the linear fractional transformation $w = \frac{2z}{3z-1}$.
   a) 1, -1
   b) 1,2
   c) I, -i
   d) 0,1
3. Which of the following is true about a non-constant analytic function?
   a) Can be purely real
   b) Can be purely imaginary
   c) Can have constant modulus
   d) None of these
4. Evaluate $\sqrt{3+4i}$
   a) $2-i$
   b) $2+i$
   c) $\pm(2+i)$
   d) $\pm(2-i)$
5. Which of the following is/are true
   a) Only real numbers have purely real or purely imaginary square root
   b) Only positive real numbers have purely real square root
   c) Only negative real numbers have purely imaginary square root
   d) All the above are true
6. A single valued analytic branch of $\log z$ can be defined on which of the following regions?
   a) Complement of negative real axis
   b) Complement of positive real axis
   c) Complement of positive imaginary axis
   d) Complement of negative imaginary axis.
7. The single valued analytic branch of $Arc\cos z$ is
   a) $\log\left(z+\sqrt{z^2-1}\right)$
   b) $i\log\left(z-\sqrt{z^2-1}\right)$
   c) $i\log\left(z+\sqrt{z^2-1}\right)$
   d) $-i\log\left(z-\sqrt{z^2+1}\right)$

8. Which of the following is an example of an indirectly conformal mapping

a) $f(z) = z^2$

b) $f(z) = e^z$

c) $f(z) = -z$

d) $f(z) = \bar{z}$

9. Which of the following is not a linear fractional transformation?

a) $S(z) = \dfrac{3z+4}{2z-2}$

b) $S(z) = \dfrac{z+2}{2z+5}$

c) $S(z) = \dfrac{3z+6}{2z+4}$

d) $S(z) = \dfrac{z+1}{2z-5}$

10. Which of the following is a normalized linear fractional transformation?

a) $S(z) = \dfrac{z+3}{z-4}$

b) $S(z) = \dfrac{3z+2}{4z+3}$

c) $S(z) = \dfrac{z+2}{2z+1}$

d) $S(z) = \dfrac{z+4}{z-1}$

11. The matrix representation of the basic linear fractional transformation inversion is

a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

12. Which of the following represents translation?

a) $\begin{bmatrix} 0 & 1 \\ 1 & \alpha \end{bmatrix}$

b) $\begin{bmatrix} \alpha & 1 \\ 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 \\ \alpha & 0 \end{bmatrix}$

13. How many linear transformations are there with $1, 0$ and ∞ are fixed points.

a) One

b) Two

c) Three

d) Four

14. Find the image of the point at infinity under the linear transformation $T(z) = \frac{4z-3}{5z+1}$

a) ∞

b) 0

c) $\frac{4}{5}$

d) $\frac{5}{4}$

15. Find the argument of the cross ratio $(1, i, -1, -i)$

a) 0

b) $\frac{\pi}{4}$

c) $\frac{\pi}{2}$

d) $\frac{\pi}{3}$

16. Which of the following is true about the cross ratio $(-1-i, 0, 1+i, 2+2i)$

a) A real number

b) Argument is $\frac{\pi}{4}$

c) Argument is $\frac{3\pi}{4}$

d) An imaginary number

17. Which of the following can be an analytic function?

a) $f(x, y) = x^2 + y^2$

b) $f(x, y) = x^2 + 2i$

c) $f(x, y) = 4 + 2(x + y)$

d) None of the above

18. The image of a straight line under a linear transformation is

a) A straight line

b) A circle or a straight line

c) An ellipse
d) A parabola

19. What is the maximum number of distinct cross ratios that can be constructed by permuting four given points?
a) 5
b) 10
c) 6
d) 24

20. If z and z^* are symmetric points with respect to a circle passing through the points z_1, z_2, z_3 then which of the following is a correct statement.
a) $(z, z_1, z_2, z_3) = (z^*, z_1, z_2, z_3)$
b) $(z, z_1, z_2, z_3) = (-z^*, z_1, z_2, z_3)$
c) $(\bar{z}, z_1, z_2, z_3) = (z^*, z_1, z_2, z_3)$
d) $(z, z_1, z_2, z_3) = \overline{(z^*, z_1, z_2, z_3)}$

21. The mapping which carries a point z into its symmetric point z^* with respect to a given circle is called
a) linear transformation
b) reflection
c) rotation
d) translation

22. Which of the following are true
a) Every linear transformation carries circles in to circles
b) Every reflection carries circles in to circles
c) Every linear transformation preserves symmetry
d) All the above

23. The linear transformation $T(z) = \frac{z}{2z-1}$ is
a) Elliptic
b) Parabolic
c) Hyperbolic
. d) Lexodromic

24. How many points at infinities are there in the complex plane
a) Infinitely many
b) Two
c) Three
d) One

25. If $f(x, y) = x^2 + 2xy + iV(x, y)$ is analytic then evaluate $\frac{\partial V}{\partial y}$
a) $x + y$
b) $x^2 + 2xy$

c) $2(x+y)$

d) $x-y$

26. What is the length of the circle with points $z = a + \delta e^{it}$, $0 \leq t \leq 2\pi$

a) $2\pi i\delta$

b) $2\pi\delta$

c) $\pi\delta^2$

d) $\pi\delta$

27. What is the length of the arc determined by the points $z = t + it$, $0 \leq t \leq 2$

a) $\sqrt{2}$

b) 4

c) $2\sqrt{2}$

d) 8

28. Evaluate the integral $\int_i^{i/2} e^{\pi z} \mathrm{dz}$

a) $(1+i)/\pi$

b) $i\pi$

c) $-2\pi i$

d) $1+i$

29. What is the value of the integral $\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) \mathrm{dz}$

a) π

b) 0

c) e+ $\frac{1}{e}$

d) πe

30. Evaluate the integral $\int_1^3 (z-2)^3 \mathrm{dz}$

a) 0

b) 1

c) 2

d) e

31. Evaluate the integral $\int_{|z|=1} \frac{dz}{z}$

a) 0

b) 1

c) $2\pi i$

d) 2π

32. What is the value of the integral $\int_{|z|=1} z^2 \mathrm{dz}$

a) 1

b) 0

c) $2\pi i$

d) 2π

33. Find the integral $\int_{|z|=2} \sin z \, dz$

a) 0
b) 1
c) π
d) 2π

34. What is the value of $\int_{|z|=1} e^z$ dz ?

a) 1
b) 0
c) e
d) π

35. Evaluate $\int_{|z|=1} \frac{e^z}{z} \, dz$

a) 0
b) 1
c) π
d) e

36. Evaluate $\int_{|z|=2} \frac{dz}{z^2+1}$

a) 0
b) 1
c) 2
d) π

37. Evaluate $\int_{|z|=2} \frac{dz}{z^2-1}$

a) 1
b) 0
c) 2
d) i

38. Evaluate $\int_{|z|=1} \frac{dz}{z-2}$

a) 0
b) 1
c) i
d) iπ

39. Evaluate $\int_{|z|=1} \frac{dz}{z(z+2)}$

a) 0
b) 1
c) $2\pi i$
d) iπ

40. Evaluate $\int_C \frac{z^2 dz}{z-3}$ where C is the unit circle

a) 0
b) 1

c) π

d) iπ

41. Let C be the unit circle. Find the value of $\int_C ze^{-z}$ dz.

a) e

b) e^2

c) 0

d) 1

42. Let $f(z) = \frac{1}{z^2+2z+2}$ and C be the unit circle. Find the value of the integral $\int_C f(z)dz$

a) 1

b) 2

c) 0

d) 2e

43. Let C be the positively oriented circle $|z - i| = 2$ and $g(z) = \frac{1}{z^2+4}$. Evaluate $\int_C g(z)dz$

a) 0

b) 1

c) $\pi/2$

d) iπ

44. Let C be the unit circle $|z - i| = 2$ in the positive sense and $f(z) = \frac{1}{z^2-1}$. Find $\int_C f(z)dz$

a) 1

b) 0

c) π

d) iπ

45. The process of finding the length of an arc is called …. .

a) differentiation

b) integration

c) rectification

d) quadrature

46. What is the antiderivative of $sech^2 z$?

a) $\sinh z$

b) $\cosh z$

c) $\tanh z$

d) sin z

47. Let γ be any simple closed curve and $f(z) = ze^z$. What is the value of $\int_\gamma f(z)\,dz$?

a) 1

b) e

c) π

d) 0

48. Let R be the rectangle bounded by the lines Re $z = \pm 1$, Im $z = \pm 1$ and $f(z) = \sin z$. Find $\int_{\partial R} f(z)\, dz$?

a) πi
b) 0
c) 1
d) 2π

49. Let γ be any simple closed curve and a be a point outside γ. Also let $f(z) = e^z$, evaluate $\int_\gamma \frac{f(z)\, dz}{z-a}$?

a) $2\pi i$
b) 1
c) 0
d) $-i\pi$

50. Let $f(z) = e^z$ and γ be the positively oriented circle $|z-1| = 2$. Evaluate $\int_\gamma \frac{f(z)\, dz}{z-1}$?

a) 1
b) 0
c) $2\pi i$
d) $2\pi i e$

51. A non constant entire function is

a) bounded
b) unbounded
c) meromorphic
d) discontinuous

52. An entire function which is bounded in the whole plane is

a) constant
b) meromorphic
c) discontinuous
d) integrable

53. $\int_\gamma f(z)\, dz = 0$ for all cycles γ in a region Ω. Then $f(z)$ is

a) constant
b) analytic in Ω
c) entire
d) bounded

54. If $f(a)$ and all derivatives $f^n(a)$ vanish, then $f(z)$ is

a) identically vanishes in the region
b) reduces to a non-zero constant

c) bounded in the whole plane
d) unbounded in the region

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.55. Evaluate $\int_{|z|=1} z^{-4} \sin z \; dz$

a) 0
b) 1
c) iπ
d) -iπ/ 3

56. Find the value of $\int_{|z|=1} \frac{e^z}{z^3} dz$.

a) 0
b) 1
c) iπ
d) 2πi

57. Find the value of $\int_{|Z|=1} z^{-2} e^{-z} \; dz$.

a) 0
b) iπ
c) 2πi
d) -2πi

58. Evaluate $\int_{|z|=1} e^z z^{-n} \; dz$.

a) 2πin
b) 2πi(n)!
c) $\frac{2\pi i}{(n-1)!}$
d) $\frac{2\pi i}{n!}$

59. $\int_{|z|=1} \frac{\cos hz}{z^4} dz$.=?.

a) 1
b) 0
c) 2πi
d) iπ

60. Find $\int_{|z-i|=2} \frac{e^z}{(z+1)^2} dz$.

a) 1
b) $\frac{2\pi i}{e}$
c) 2πie
d) $\frac{i\pi}{e}$

61. $\int_{|2|} \frac{e^{2z}}{z^4}\, dz =$?

a) $\frac{2\pi i}{3}$
b) $\frac{8\pi i}{3}$
c) $\frac{i\pi}{3}$
d) $\frac{-i\pi}{3}$

62. $f(z) = \frac{\sin z}{z}$ has a removable singularity at

a) $z = 1$
b) $z = -1$
c) $z = 0$
d) $z = \infty$

63. Identify the nature of the singularity of $f(Z) = \frac{\sin z}{z}$ at the origin

a) pole
b) removable singularity
c) essential singularity
d) zero

64. $z = 0$ is a pole of the function

a) $f(z) = \frac{e^z}{z}$
b) $f(z) = \frac{\sin z}{z}$
c) $f(z) = e^{1/z}$
d) $f(z) = z \sin\left(\frac{1}{z}\right)$

65. $z = 1$ is a r singularity of

a) $\frac{\sin z}{z}$
b) $\frac{\sin (z-1)}{(z-1)}$
c) e^{z-1}
d) $\sin(z-1)$

66. $z = 2$ is a double pole of the function

a) $z^2 e^z$
b) $\frac{e^z}{z^2}$
c) $\frac{e^z}{z(z+1)}$
d) $\frac{e^z}{z^3}$

67. $f(z) = e^{1/z}$ has an essential singularity at

a) z = 1
b) z = 2
c) z= -1
d) z = 0

68. $f(z) = \sin\left(\frac{1}{z-1}\right)$ has an essential singularity at

a) z = 1
b) z= 0
c) z = -1
d) z = ∞

69. $f(z) = \frac{e^{1/z}}{(z-1)}$ has an essential singularity at

a) z = 0
b) z= 1
c) z = -1
d) z = 2

70. $f(z) = \frac{e^{1/z}}{(z+1)}$ has a simple pole at

a) z = 0
b) z= 1
c) z = -1
d) z = ∞

71. $f(z) = \frac{1}{\sin \pi z}$ has a non isolated essential singularity at

a) z = 0
b) z = 1
c) z = -1
d) z = 2

72. Evaluate $\int_{|z|=2} \frac{(z-2)}{(z^2-4z+3)} dz$.

a) 1
b) 2
c) iπ
d) 2πi

73. Find the value of $\int_{|z|=2} \frac{2z-1}{z^2-z} dz$.

a) 0
b) iπ
c) 2πi
d) 4πi

74. Find the value of $\int_{|z|=2} \frac{z}{z^2+1}\, dz$.

a) iπ
b) 2πi
c) - iπ
d) – 2πi

75. Find the value of $\int_{|z|=1} \frac{3z^2-4}{z^3-4z}\, dz$.

a) iπ
b) – iπ
c) 2πi
d) 0

76. Which of the following properties do not change the identity of a chain of arcs:
a) Permutation of two arcs
b) Subdivision of an arc
c) Cancellation of opposite arcs
d) All of the above

77. An integral along a simple closed curve is called a ………… .
a) Multiple integral
b) Jordan integral
c) Contour integral
d) None of the above

78. A region which is not simply connected is called ……………… region.
a) Multiple curve
b) Jordan connected
c) Connected curve
d) Multiply connected

79. For all cycles γ in Ω and all points a which do not belong to Ω, the region Ω is simply connected if and only if …………….. .
a) $n(\gamma, a) < 0$
b) $n(\gamma, a) = 0$
c) $n(\gamma, a) > 0$
d) None of the above

80. If f(z) is analytic and f'(z) is continuous at all points inside and on a simple closed curve γ, then
a) $\oint_{\gamma} f(z)dz = 0$
b) $\oint_{\gamma} f(z)dz \neq 0$
c) $\oint_{\gamma} f(z)dz = 1$
d) $\oint_{\gamma} f(z)dz \neq 1$

81. If f(z) is analytic and f'(z) is continuous at all points in the region bounded by the simple closed curves γ_1 and γ_2, then
a) $\oint_{\gamma_1} f(z)dz = \oint_{\gamma_2} f(z)dz$
b) $\oint_{\gamma_1} f(z)dz \neq \oint_{\gamma_2} f(z)dz$

c) $\oint_{\gamma_1} f(z)dz = \oint_{\gamma_2} f'(z)dz$

d) $\oint_{\gamma_1} f(z)dz \neq \oint_{\gamma_2} f'(z)dz$

82. A point z_0 at which a function f(z) is not analytic is known as a …………….. of f(z).

a) Residue
b) Singularity
c) Winding number
d) None of these

83. The value of $\oint_C \frac{1}{z^2+4} dz$ where C is the circle |z-2i|=1 will be:

a) 0
b) 1/5
c) $\frac{\pi}{2}$
d) $\frac{\pi}{3}$

84. Let $f(z) = \frac{1}{z^2+6z+9}$ defined in the complex plane. The integral $\oint_C f(z)dz$ over the contour of a circle with centre at the origin and unit radius is …………… .

a) 3π
b) $\pi i/2$
c) 0
d) 1

85. The value of the integral $\oint_C \frac{z+1}{z^2-4} dz$ in counter clockwise direction around a circle C of radius 1 with center at the point z=-2 is …….. .

a) $\pi i/2$
b) $2\pi i$
c) $-\pi i/2$
d) $-2\pi i$

86. The value of the integral $\oint_C Cos\, 2\pi z\, dz$, where C: |z|=1 is

a) $2\pi i$
b) $4\pi i$
c) $-2\pi i$
d) 0

87. If $p\, dx + q\, dy$ is locally exact in a simply connected region Ω, then for every cycle $\gamma \sim 0$ in Ω,

a) $\oint_\gamma p\, dx + q\, dy \neq 0$
b) $\oint_\gamma p\, dx + q\, dy = 0$
c) $\oint_\gamma p\, dx + q\, dy \geq 0$
d) $\oint_\gamma p\, dx + q\, dy \leq 0$

88. The single valued analytic branch of $\log z$ can be defined in any simply connected region which does not contain ………….. .

a) The positive real axis
b) The negative real axis
c) The origin
d) None of the above

89. The poles of $f(z) = \frac{z-2}{z^2}\sin\left(\frac{1}{z-1}\right)$ is ……. .

a) 2
b) 0
c) 1
d) None of these

90. The poles of $f(z) = \frac{z^2+1}{1-z^2}$ is ……… .

a) 1
b) -1
c) ± 1
d) 0

91. The residue of $f(z) = \cot z$ at each poles is …… .

a) 0
b) 1
c) ½
d) None of these

92. If $f(a) = 0$ and $f'(a) \neq 0$, then $z = a$ is called a ……….. .

a) Simple zero
b) Simple curve
c) Zero of order n
d) None of these

93. An integral of the form $\int_{-\infty}^{\infty} R(x)dx$ converges if and only if in the rational function R(x)

a) The degree of the denominator is at least one unit higher than the degree of the numerator and if one pole lies on the real axis
b) The degree of the denominator is at least two units higher than the degree of the numerator and if no pole lies on the real axis
c) The degree of the denominator is at least two units higher than the degree of the numerator and if no pole lies on the imaginary axis
d) The degree of the denominator is at least one units higher than the degree of the numerator and if no pole lies on the imaginary axis

94. How many roots of the equation $z^4 - 6z + 3 = 0$ have their modulus between 1 and 2 ?

a) 0
b) 1
c) 2
d) 3

95. The residue of $f(z)$ at an isolated singularity a is the unique complex number R which makes ………………… the derivative of a single valued analytic function in an annulus $0 < |z-a| < \delta$.

a) $f(z) - \frac{R}{z-a}$
b) $f(z) - \frac{R^2}{z-a}$
c) $f(z) - \frac{R}{(z-a)^2}$
d) None of these

96. If $f(z)$ is analytic in a region Ω, then for every cycle γ which I homologous to zero in Ω, $\frac{1}{2\pi i}\int_{\gamma} \frac{f(z)\,dz}{z-a}$ =…………….. .

   a) $n(\gamma, a)f(a)$
   b) $n(\gamma, a)f'(a)$
   c) $n(\gamma, a)f''(a)$
   d) None of these

97. If $n(\gamma, a)$ is defined and equal to 1 for all points $a \in \Omega$ and either undefined or equal to zero for all points a not in Ω. Then the cycle γ is said to ………… the region Ω.

   a) Wind
   b) Unbound
   c) Bound
   d) None of the above

98. If $f(z)$ is meromorphic in a region Ω with the zeros a_j and the poles b_k, then for every cycle γ which is homologous to zero in Ω and does not pass through any of the zeros or poles

$$\frac{1}{2\pi i}\int_{\gamma} \frac{f'(z)}{f(z)}\,dz = \sum_j n(\gamma, a_j) - \sum_k n(\gamma, b_k)$$

   The above stated theorem is:

   a) Rouche’s theorem
   b) Argument principle
   c) Residue theorem
   d) None of the above

99. To find the number of zeros of an analytic function $f(z)$ in the disc $|z| \leq R$, we can use the following theorem:

   a) Residue theorem
   b) Cauchy’s integral theorem
   c) Rouche’s theorem
   d) None of the above

100. The integral of an exact differential over any cycle is ….

   a) Zero
   b) Infinite
   c) Not defined
   d) A finite number other than zero